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Journal
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THE LUMINOUS PROPERTIES OF THE BLACK BODY

By Herbert E. Ives

ABSTRACT

The latest values for the constants of the black body are used, in conjunction with the Gibson-Tyndall luminous efficiency curve of the spectrum, to compute the luminous efficiency and brightness of the black body over a large range of temperatures. The value .00161 watts per lumen is used for the least mechanical equivalent of light. The computed values agree everywhere to within one per cent with the most recent experimental values.

The computation of the luminous properties of the black body provides an extremely sensitive test of the accuracy and consistency of the various constants of radiation and illumination which are involved. The radiational constants involved are: 1. The fixed points of the high temperature scale; 2. The Stefan-Boltzmann radiation constant (σ); 3. The Wien-Planck constant (c_2). The latter constants are themselves, in the light of recent physical theory, derivable from more fundamental constants of physics which may be obtained from a variety of experiments. The constants of illumination concerned are: 1. The luminous efficiency distribution by wave length through the spectrum; 2. The least mechanical equivalent of light.

The earlier attempts to compute the luminous characteristics of the black body, for instance candle power per unit area, led to results which differed by as much as 30 or 40 per cent from observed values.[1] The causes of these discrepancies were various. The photometric methods employed in the measurement of the luminous properties were not consistent with the luminous efficiency curves of the spectrum employed; the luminous efficiency curves were not determined with anything approaching sufficient accuracy; and the values used for the

[1] For references covering this whole subject, see Ives, The Photometric Scale, Jour. Franklin Institute, *188*, p. 217; 1919.

least mechanical equivalent of light were greatly in error. Curious as it may seem to some, it is nevertheless true that the most persistent source of disagreement between computed and observed values for this test case were uncertainties in the radiational constants. This is because the determination of temperatures by optical pyrometric methods, which are alone available in the temperature regions of most interest, and the computation of radiant output for the short wave length visible region, vary as a high order of the value adopted for the constant c_2. Thus in the temperature region of most interest, an uncertainty of one part in the fourth place of this constant is sufficient to prevent agreement of computed and observed values by an amount greater than is permissible in photometric measurement. It has consequently been chiefly due to the greater accuracy of the determinations of the radiational constants in recent years, that the problem under consideration has been brought to a reasonably satisfactory solution.

The data which it is the purpose of this paper to present were computed for the International Critical Tables. In making these computations, the most recent values of the radiational constants, as selected for use in these tables, were taken. They are, for σ, 5.709×10^{-12}; for c_2, 14330. For the luminous constants involved, the most recent determination of the spectral luminous efficiency curve, namely that of Gibson and Tyndall,[2] was employed, as expressed in the four term equation of the same experimenters.[3] For the least mechanical equivalent of light, the value .00161 watts per lumen was used. This value—which is practically identical with that obtained by a very direct method of determination by the writer and his co-workers ten years

TABLE 1

Authors	Value as Published	Value as Recomputed
Ives, Coblentz, and Kingsbury[4] (λ5461)	.00159	.001605
Holst and Visser[5] (BB. 1336°)	.00159 to .00168	.00161
Hyde, Forsythe, and Cady[6] (BB. 2100°)	.00150	.001615
Ives[7] (BB. 2035°)	.00161	.001605

[2] Gibson and Tyndall, B. S. Sci. Paper, 475; 1923.
[3] Tyndall and Gibson, J. O. S. A. & R. S. I. *9*, p. 403; 1924.
[4] Ives, Coblentz, and Kingsbury, Phys. Rev. *5*, p. 269; 1915 and *6*, p. 319; 1915.
[5] Holst and Visser, Proc. Kon. Akad. Amsterdam *20*, p. 1036; 1918.
[6] Hyde, Forsythe, and Cady, Phys. Rev. *13*, p. 45; 1919.
[7] Ives, J. O. S. A. & R. S. I. *9*, p. 635; 1924.

TABLE 2. *Black body luminous characteristics*

T, Degrees Absolute	Radiant Luminous Efficiency	Brightness, *b* in Candles per cm² Computed for $m = .00161$	Brightness, *b* in Candles per cm² Observed, Hyde Forsythe and Cady
1200	0.00000602	0.0141	
1400	0.0000557	0.242	
1600	0.000282	2.08	
1700	0.000541	5.10	5.0_6
1750	0.000726	7.69	7.6_9
1800	0.000957	11.3	11.2
1850	0.00124	16.4	16.2
1900	0.00158	23.2	23.0
1950	0.00198	32.3	32.0
2000	0.00246	44.4	44.0
2050	0.00301	59.9	59.6
2100	0.00364	79.8	79.6
2150	0.00436	105.	104.9
2200	0.00517	137.	136.4
2250	0.00606	175.	175.
2300	0.00706	223.	222.
2350	0.00816	281.	280.
2400	0.00935	350.	350.
2450	0.0107	433.	433.
2500	0.0120	531.	527.
2550	0.0135	645.	641.
2600	0.0151	780.	775.
2650	0.0168	934.	927.
3000	0.0309	2.83×10^3	
4000	0.0807	2.33×10^4	
5000	0.1190	8.40×10^4	
6000	0.1353	1.98×10^5	
7000	0.1352	3.67×10^5	
8000	0.1258	5.82×10^5	
10000	0.0987	1.115×10^6	

ago—was chosen on the basis of a review for the International Critical Tables computation of the chief recent determinations of this constant, as re-calculated to the latest values of the radiational constants, and of the luminous efficiency curve of the spectrum. A summary of the least mechanical equivalent values, both as originally published and as re-calculated, is given in Table 1.

The methods of computation have previously been very fully discussed[1] and need not be repeated here. Using these methods, the results for the radiant luminous efficiency, and the brightness, in candles per square centimeter, for black body temperatures between

1200° and 10,000° absolute have been computed and are given in Table 2. The last column of the table gives the brightnesses at the corresponding temperatures as directly observed in an extended series by Hyde, Forsythe, and Cady.[6] These latter values have been recomputed to the temperature scale fixed by the value of 14330 for the constant c_2, in place of the figure 14350 originally used, on the basis of data kindly furnished by Dr. Forsythe.

Comparing the observed and computed values, it will be found that these nowhere differ by as much as 1 per cent; in short, the agreement is of the order of attainable photometric accuracy under the conditions involved, and indicates a very satisfactory agreement between the various sets of data employed in the computations. For the temperature region near 2100°K, where no questions of the heterochromatic photometric methods used arise, the agreement is within 1/2 per cent. It may be remarked that the agreement between the observed and computed values might be made to average even closer (<0.5 per cent) over the whole temperature region tabulated if a slightly higher value for the least mechanical equivalent were used, namely .00162. It has, however, seemed advisable on the whole to present the data as they stand, leaving for the future such further progress toward absolute agreement as may be expected to come from complete standardization of heterochromatic photometric methods, and the exact determination of higher fixed points on the temperature scale.

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Sub-electrons.—This interminable controversy shows as yet no signs of approaching its predestined end. Daecke's contribution consists in a profound statistical investigation of 24 values of charge inferior to e obtained by Kœnig with mercury droplets. He concludes that there is a peculiar tendency of these values to group themselves about integer multiples of the quantities $e/19$, $e/9$, $e/7$, and $e/5$; further, that there are distinct signs of a tendency to grouping around integer multiples of $e/16$, $e/13$, $e/12$, $2e/21$, $e/6$, $e/4$, $2e/5$, and $e/2$. The inference is that the subelectronic charges, whether real or only simulated, tend to be equal to me/n, in which m and n denote integers, of which n is "not large" and $m<n$. Without venturing to impugn the theorem on which these conclusions rest, one may feel that it is rather odd that a mere twenty-four experimental values should group themselves about integer multiples of such a multitude of different numbers. [H. Daecke, Hamburg: Phil. Mag. *50*, pp. 637-644; 1925.]

KARL K. DARROW

THE CHROMATIC ABERRATION OF THE EYE, THE CHROMATIC VARIATIONS IN THE INTERVAL OF STURM, AND ALLIED PHENOMENA AS DETERMINED BY A SUBJECTIVE METHOD OF SKIASCOPY*

By Charles Sheard

From the time of Newton[1,2,3] down to the present day investigators of the science of light have been interested in the aberrations of the human eye. The reasons for this interest have been varied. In some cases the investigations have been undertaken from the standpoint of pure optical science only, and wholly with the desire to study and determine physical constants, thereby considering the eye and its aberrations in the same manner as one would any optical instrument or system of lenses. And again, in a few instances, notably that of Ames, Proctor, and Ames[4], the object has been to apply the experimental determinations and deductions to the interpretation of that which is external to the eye, or the external world as it is imaged on the retina, and to endeavor to discover, by the processes of painstaking experimentation, the characteristics of retinal images as to form, color, accentuation of line, and chromatic edges in order that the artist's pictorial representation of Nature may be similar to the general characteristics of the pictures (or receptor impressions) received at the retina. As is well known, these most important characteristics are spherical aberration, chromatic aberration, oblique astigmatism, and distortion.

I have made brief reference to the two outstanding reasons for the various investigations which have been carried out relative to the aberrational errors of the eye. There is still a third field in which a study of these aberrations may be of value. I refer to that branch of applied

* Presented at the meeting of the Optical Society of America, October 24, 1924. An abstract of this paper appeared in J. O. S. A. & R. S. I. *10*, pp. 292-294; 1925.

[1] Helmholtz: Chromatic aberration in the eye. Physiol. Optics, Optical Society of America, *1*, pp. 172-178; 1924.

[2] Sheard, Charles: Physiological optics. Chicago, Cleveland Press, pp. 110-116; 1918.

[3] Tscherning: English translation. Physiol. Optics, pp. 108-114; 1904.

[4] Ames, Proctor and Ames: Vision and the technique of art. Proc. Am. Acad. Arts and Sc. *58*, pp. 3-47; 1923.

science technically designated as ocular refraction, having to do with applications of the laws and principles of physiologic optics to the detection and correction of anomalies of refraction, accommodation, and convergence. This paper, therefore, is an account of investigations undertaken to determine the variations in the chromatic aberration of an eye by methods which are properly classified in the domain of ocular refraction, and is, furthermore, a presentation of experimental facts and deductions, drawn from these investigations, which should prove of interest and of value to the refractionist. By reason of the fact, however, that the subject must be treated necessarily from the three viewpoints which have been previously stated, it seems possible that this contribution may prove of interest, in some particulars at least, to all those who are in any way interested in human vision.

HISTORICAL SURVEY AND DEFINITIVE STATEMENTS

Being appreciative of the fact that many may not be familiar with the terms and nomenclature which will be used, I shall present a brief summary of some of the commonly accepted data and statements relative to ocular chromatic aberration, as well as certain definitions of terms employed in setting forth the results of the investigations herein reported (Figs. 1 and 2).

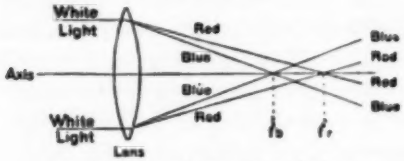

Fig. 1. *Chromatic aberration of a lens.*

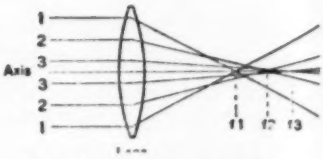

Fig. 2. *Spherical aberration of a lens.*

Young made the first estimates of the chromatic aberration of the eye, while Fraunhofer made the first reliable measurements of the difference in the focal lengths of the eye for the extreme spectral colors. Observing a prismatic spectrum through an achromatic telescope, the eye-piece of which carried a cross-hair, he found that an eye which saw, without accommodation, an object of color corresponding to the spectral line C, could not see that object clearly, with the same accommodative status, 18 to 24 inches nearer the eye. Helmholtz modified this experiment by allowing light from various portions of the spectrum to pass through a very small opening in a screen and determining the greatest distance at which this opening still remained of pin point size. From the data obtained Fraunhofer found a chromatic aberration of from 1.5 to 3 diopters, and Helmholtz of 1.8 diopters.

The methods followed in the more recent work of Nutting[5] and of Ames and Proctor[6,7] are fundamentally the same. The procedure consists in determining quantitatively the axial chromatism of a lens by ascertaining the difference in the inclination of rays of varying wave lengths which come to the same focus. Nutting found the mean axial chromatism of seven eyes between the C and F lines to be 0.99 diopters, and the results of Ames and Proctor on different subjects show amounts varying from 0.72 diopter to 0.98 diopter. One vital point of difference exists in the character of the chromatic aberration wave length curves as obtained by these two sets of investigators. Nutting found that in most cases the chromatic aberration wave length curve was appreciably flattened throughout its central portion, that is, on both sides of the sodium D line, and therefore concluded that the eye is more or less chromatically corrected. Ames and Proctor, however, found no such flattening, and therefore disagree with Nutting and attempt, although apparently unsuccessfully, to account for the disparity in the two sets of experimental results. In the data and curves which are presented in this paper there will be found chromatic aberration wave length relations for the same eye which are in agreement with both of the foregoing conclusions. The character of these curves varies, however, with the fixation distance. It is hoped that the causes of these very considerable differences have been determined and disclosed in this paper.

Since the refractive indexes of the optical media of the eye do not differ greatly from the index of water, it might seem as though the chromatic aberration wave length curve for the human eye should follow approximately that for water. The results of Ames and Proctor and of Nutting indicate that such is only roughly the case. By reason of the experimental procedure which I have adopted I have found that the form of the chromatic aberration wave length curve is quite similar to that of the refractive index wave length curve of water in certain experimental determinations and, again, in other instances, quite dissimilar, definitely indicating that the distance of fixation, size of pupil, spherical aberration, and possibly other factors, have marked influence on the forms of these chromatic aberration curves.

In none of the investigations with which I am acquainted are there any quantitative experiments dealing with the effects of (1) distance of fixation, that is with a non-accommodated eye and, again, in various

[5] Nutting, P. G.: The axial chromatic aberration of the human eye. Proc. Roy. Soc. London, *90*, pp. 440-443; 1914.

[6] Ames, Proctor: Aberrations of the eye. *J. O. S. A.*, 22, p. 84; 1921.

[7] Ames, Proctor: Aberrations of the eye. Am. Jour. Physiol. Optics, *4*, 3-37; 1923.

states of accommodation; (2) size of pupil or artificial diaphragm, and (3) variations in amounts of chromatic aberration and shapes of chromatic aberration wave length curves. The major portion of this paper is devoted to a presentation of data and discussion of these effects. In the face of the data obtained, I still feel that much needs to be more definitely known before experimental results can be completely and properly interpreted and the results of various investigators coördinated. I also recognize that the results obtained in an investigation involving one or two eyes only cannot be accepted as necessarily conclusive. It is probable that every eye has its individual characteristics relative to spherical and chromatic aberrations, oblique astigmatism and distortion. However, some data given by us Luckiesh on what he calls retiring and advancing colors indicate that the character and magnitude of the effects in which we are interested are sufficiently constant to warrant the belief that conclusions and deductions from experimentation on one pair of eyes only would be quite generally applicable. Luckiesh[8] found, however, two eyes, out of the nine or ten subjected to his tests, in which the results were negative on the basis of positive relations existing between the location of a red *X* and blue *E* as used in his experimentation.

In order to compare the results of my work with those given by investigators who have reported data in linear distances in front of or behind the retina instead of making use of the dioptric system, the statement that 1 mm change in depth of the eye or position of the image with reference to the retina corresponds to a difference of 2.5 diopters may be accepted as sufficiently accurate for our purposes. I believe, however, that there is an advantage in methods which involve dioptric measurements of chromatic differences rather than linear distances, although the latter methods, other things being equal, are doubtless more accurate because trial case lenses are not made in units closer than 0.12 diopter. The methods which were followed in the present investigations afford data comparatively easily and quickly and I believe that they give us information which is as valuable as that obtained by more elaborate experimental equipment and more laborious and tedious experimental procedures.

THE SUBJECTIVE METHOD OF SKIASCOPY OR SKIAMETRY

The term skiascopy means the measurement of a shadow. In ophthalmologic, optometric, and optical literature one will find the words skias-

[8] Luckiesh, M.: On retiring and advancing colors. Am. Jour. Psychol., *29*, pp. 182-186; 1918.

copy, skiametry or retinoscopy rather indiscriminately used to signify the objective method of determining the state of refraction of an eye by illuminating it with light thrown from a mirror (usually a plano-mirror, although it may be concave), containing a small hole or non-silvered area through which the observer regards the direction of movement of the retinal illumination and its bordering shadows, as seen in the observed pupil, when the mirror is rotated. When light is thrown into an eye, as for example by means of a mirror, the fundus is illuminated. By looking through the sight-hole of the mirror an observer will receive light coming back from the illuminated portions (red fundal reflex) of the eye under observation, and also will perceive the shadow bordering this area. When the mirror is rotated the illuminated area and shadow will move across the observed pupil. The observer ultimately places

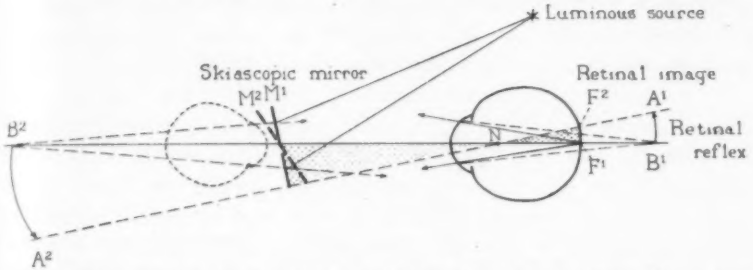

Fig. 3. *Diagram illustrating the principles of objective skiametry in a condition of hyperopia.*

before the eye under test such lenses as will cause all apparent movement of light coming from the illuminated retinal area to cease; hence the pupil will appear filled with light, there being no shadow apparent to the observing eye as the mirror is rotated. In other words, by means of lenses the observer endeavors to bring the reversal or neutral point of the observed eye to the nodal point of his own eye. This procedure can be readily carried out in practice and the refractive error of an eye objectively determined. A movement of the shadow in the same direction as the movement or rotation of a plano-mirror, due allowance being made in dioptric equivalence for the "working distance" of the observer, indicates an hyperopic correction, while a movement in the opposite direction signifies myopia. The essential principles of skiascopy are graphically illustrated in Figs. 3 and 4.

In the objective method of skiascopy there are, therefore, these fundamental features: (1) a source of external illumination, generally obtained as a reflection of light from a mirror which can be readily ro-

tated, (2) a movement of an illuminated area on the retina followed by a shadow or area of non-illumination, (3) the interception of the light returned from the illuminated retinal area as received by the observer at the sight-hole in the mirror, and (4) the observation of the direction of movement of the illuminated area (or shadow) in the pupil of the eye under observation as compared to the direction of movement or rotation of the mirror.

In the subjective method of skiascopy, or autoskiascopy as it may be termed, these same fundamental features are present. The subject himself, however, determines the direction of movement of the shadow cast on the retina by the use of an object which is moved across his line of vision and held close to the eye under test while he is fixating a definite source of light which serves as the fixation object. Anyone can determine quite readily the general refractive condition of his own eyes by

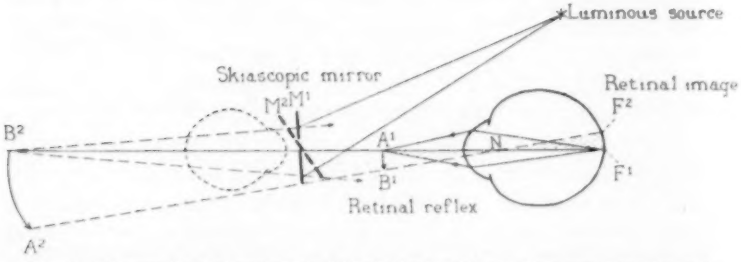

Fig. 4. *Diagram illustrating the principle of objective skiametry in a condition of myopia.*

slowly moving, back and forth before the eye being tested (the other being occluded) some object, such as a hat-pin, while looking at an incandescent lamp or other concentrated luminous source. If the eye is hyperopic, a shadow of the pin or moving pointer will appear to travel across the source of illumination in a direction opposite to that of the movement of the object. This movement of shadow will be in the same direction as that of the object in a condition of myopia.

Some years ago I observed these phenomena and tried out the method with students as an exact refractive procedure. It was found that the method required keener observation than most persons possessed, or else it was too difficult to get the persons under examination to understand the significance of the tests. This same statement, in a general way, holds true today. No published record of these observations was made, however, since I considered that they were one form only of entoptic phenomena well-known to students of physiologic optics or.

again, that they were a modification of the experimental methods adopted over a hundred years ago by Thomas Young in his studies on the spherical aberration of the human eye. My interest was revived in this method and its possibilities as an experimental rather than clinical procedure by an article from Trantas[9] under the heading of "velonoskiascopy." Trantas gives a brief statement of the method and emphasizes its applications to the determination of both the axis and the amount of astigmatism. Polack[10] says that the method is capable of a precision of 0.12 diopter. With this statement I am in agreement, since I have been able to check findings, under any specified set of fixation conditions, to within from 0.12 to 0.25 diopter and that, even when the tests were made days or even weeks apart. In 1922 Brown[11] made further refinements in methods, particularly as to the size of the fixation source of light and the use of stenopaic slits and perforations of 0.5 mm in an opaque disc, which serves as the moving object. Brown recommends a luminous screen with a slit not wider than 3 mm. By entirely different procedures, I have arrived at the same general conclusions.

Following the presentation of this article before the Optical Society in 1924, J. N. Smith[12] published a paper on "The uses of subjective retinoscopy" in which he described and illustrated a form of autoretinoscope devised by him and used in his refracting room. He discussed ways in which methods of subjective retinoscopy and autoretinoscopes may be applied to the checking of findings made by other methods, especially in difficult refractive cases.

APPARATUS AND METHODS OF OBTAINING DATA

Figs. 5 and 6 diagrammatically represent the retinal shadows and the movements of these shadows as seen in the fixation source of light when a screen is moved close to the eye and thus intercepts the light entering the eye. The diagrams are necessarily exaggerated in certain particulars in order to illustrate the *modus operandi* of this subjective method of skiascopy. The conditions obtaining for both hyperopia and myopia are illustrated. In hyperopia, if the accommodation is inoperative, the

[9] Trantas: La velonoskiascopie et son utilité surtout pour la determination des principaux méridiens de l'astigmie. Ann. d'ocul., *158*, p. 458; 1921.

[10] Polack, M. A.: Optique physiologique—Les effets du chromatisme de l'œil dans la vision des couleurs complexes. Ann. d'ocul., *158*, pp. 599-601; 1921.

[11] Brown, E. J.: Velonskiascopy in the determination of principal meridians of astigmatism. Am. Jour. Ophth., *5*, pp. 897-898; 1922.

[12] Smith, J. N.: The uses of subjective retinoscopy. Optic. Jour. and Rev., *54*, pp. 29-33; 1924.

light entering the eye focuses behind the retina, while in myopia it focuses in front of the retina. In myopia, therefore, the converging cone of light proceeding into the eye comes to its apex, or crosses, before reaching the retina; in hyperopia it does not. Obviously, then, the directions of movement of retinal shadows, and spatial shadows as seen by the observing eye, will be reversed in the two conditions. In each of the diagrams (Figs. 5 and 6) AB represents the test object, for example, wire or straight-edge, moving in a plane parallel to a vertical plane tan-

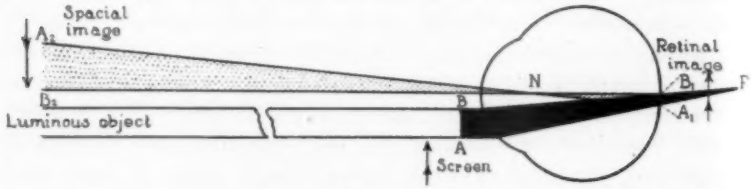

Fig. 5. *Diagram illustrating the principles of subjective skiametry in a condition of hyperopia.*

gent to the two eyes; A_1B_1 is the retinal image and A_2B_2 represents the spatial image and therefore the shadow as seen by the eye fixating the source of light. The arrows attached to the drawings indicate the directions of movements of the shadows as compared to the direction of movement of the object.

Figs. 7 and 8 show the simple pieces of equipment used for housing the source of light and for carrying various fixation screens and other accessories in the experimental work. A concentrated filament Mazda

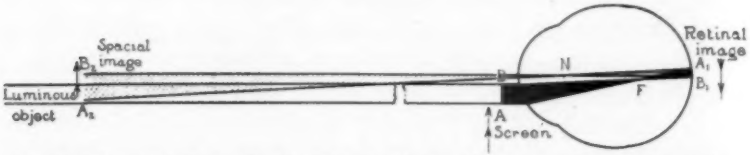

Fig. 6. *Diagram illustrating the principles of subjective skiametry in a condition of myopia.*

40-60 watt lamp was used, and a piece of ground glass was placed between the lamp and the metallic plate which was situated in the front or door of the fixation source of light. The metallic plates were made of uniform size so that they could be interchanged, and carried various sizes and shapes of apertures. The most satisfactory and generally useful piece of fixation apparatus was made by building into the door of the box a rotating sleeve into which a plate with a slit about 50 mm by 3 mm was inserted. By means of a graduated circle fitting close to the

revolvable slit it was possible to set the slit at any angle desired. This facilitates work in getting the axis and amount of astigmatism. The light filters, to which further reference will be made, were held by means of suitable supports against the face of the plate containing the slit or other aperture. The two forms of fixation apparatus, shown in Figs. 7 and 8, were painted black and mounted on backgrounds which were also blackened. Apparatus and backgrounds (generally 2 by 2 ft.) could be set at any position desired; in general the height of the slit from the floor was the same as that of the observing eye. Most of the experiments were conducted in a much darkened room. It was found that all ar-

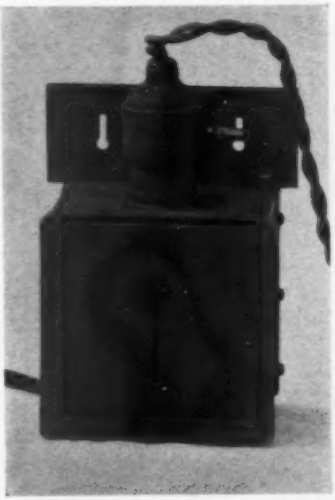

Fig. 7. *Simple form of apparatus used as a source of fixation.*

rangements which obviated the presence of extraneous light, especially in the vicinity of the source of fixation, were conducive to greater ease of observation and accuracy of judgment. Especially was this true when the observation distance was from 10 to 20 ft. or more and when refractive determinations were made with screens giving filtered light with maximal transmissions at about 480 and 720 mμ respectively.

In making the observations and in determining the refractive corrections under varying conditions of fixation distance and light filters, it became evident that the visibility of the spatial shadow was affected by: (1) the intensity of illumination of the fixation source of light, (2)

the illumination of the testing-room, (3) the character and size of the fixation object, and (4) the character and size of the moving or testing object. Comment has already been made on the first two conditions.

As a fixation object, I have tried various sizes and shapes of openings. A circular aperture, about 5 mm in diameter, generally proved more satisfactory than larger openings. The slit, about 50 mm by 3 mm, cut in a plate which was inserted in a revolving sleeve so as to enable one to place the aperture at any desired angle (as already described) proved

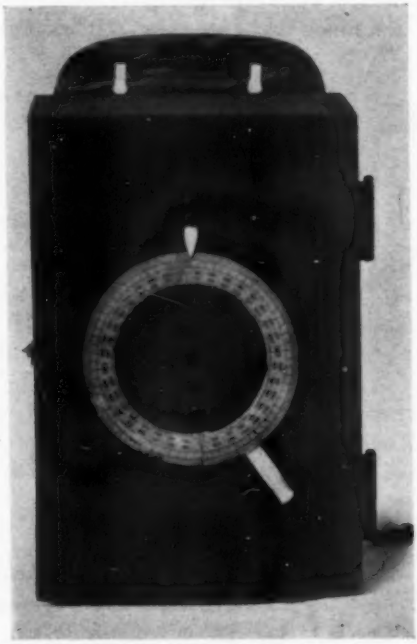

Fig. 8. *Apparatus with rotating sleeve, permitting the slit to be set at any angular position desired.*

very serviceable and was used in several experiments, especially in making observations on those in whom astigmatism was present at an angle other than 90° or 180°. Another form of fixation object, which is especially useful in testing out the principal meridians and the amount of astigmatism, is shown in Fig. 8; in it there are alternate, equal-sized and equally spaced apertures and closed blocks.

As a moving or testing object the following simple pieces of apparatus were tried: (1) a card with a straight edge, held in the hand and passed back and forth before the eye under test at such a rate as to enable the

shadow to be seen and its direction of movement determined; (2) a wire from 0.25 mm to 0.5 mm in diameter, fastened in a suitable holder so that it could be moved in any meridian desired; (3) a stenopaic slit, and (4) pin-hole or small aperture. Brown prefers the last two, but I have found a wire of small diameter most generally satisfactory. Also, from the data on the effects of diaphragms, it would not appear to be proper to use either stenopaic slits or pin-hole discs, especially if investigations are made at relatively short fixation distances. With a fixation distance of 20 ft., which is taken as the very close equivalent of infinite distance for refractive purposes, my findings indicate that the use of stenopaic slits and opaque discs with a 1 mm hole would give the closest determination of the refraction of an eye, since it would make possible the refraction along the line of vision, as limited by what is known as the visual zone or, in other words, the central refraction in contradistinction to the peripheral refraction or, perhaps, a combination of both. It is likely, therefore, that Brown's conclusions as to the size of aperture of the disc to be used for determining the amount of refractive correction when the eye is in a static condition are correct.

In obtaining the chromatic aberration curves of an eye, that is, the relations existing between the refractive corrections and the wave lengths of spectral light used in the fixation object, I have employed combinations of Wratten filters. To be sure, such combinations do not give pure monochromatic radiation, but the spectral regions have been sufficiently narrow, covering at most 30 mμ, to lead to the belief that the results are not thereby invalidated. It would also have been advantageous and would have permitted of more accurate determination of the slopes of the curves in the region 650 to 700 mμ if a filter had been available which gave a selective transmission in this region. The results detailed in this paper may, therefore, be criticized for lack of monochromaticity of illumination of the fixation object and also for possible inexactness in the shapes of the chromatic aberration curves in the region from 650 to 700 mμ. The maximal wave lengths of the selected spectral regions used in these investigations were 470, 500, 560, 620, and 720 mμ respectively.

CHROMATIC ABERRATION OF THE EYE

The results obtained from a study of the chromatic aberration of my own right eye are given in Figs. 9 to 16 inclusive. Each of the figures contains two curves which were taken for each fixation distance because of the presence of hyperopic astigmatism of about 0.75 diopter. Since

the principal refractive meridians in this case are at 90° and 180° respectively, the two curves given in each group are marked in a manner to distinguish the horizontal meridian from the vertical meridian. The terms horizontal and vertical, while referring in the charts to the meridians of the eye tested, also indicate the directions of movement

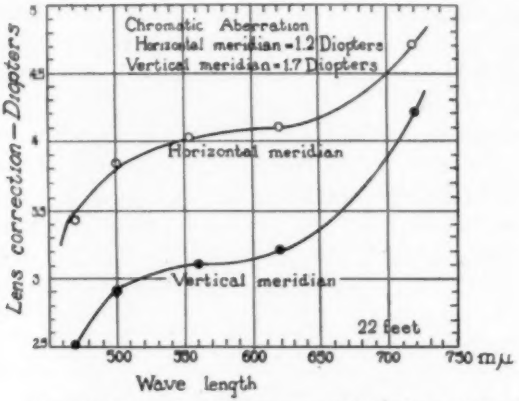

Fig. 9. *Chromatic aberration curves with fixation at 22 feet.*

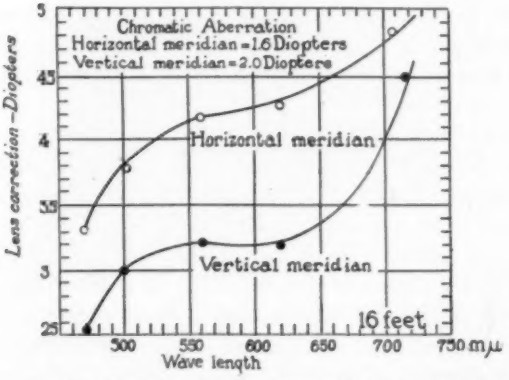

Fig. 10. *Chromatic aberration curves with fixation at 16 feet.*

of the wire or other testing object. For example, if the refractive power of the horizontal meridian of the eye was under test, the illuminated fixation slit was set vertical and the testing object was held vertically and moved horizontally in front of the pupil. The two chromatic aberration wave length curves were taken in each instance with

the idea in mind that there possibly might be sufficient difference in their shapes to account for the two types of curves obtained by Ames and Proctor, and by Nutting. Such does not appear to be the case,

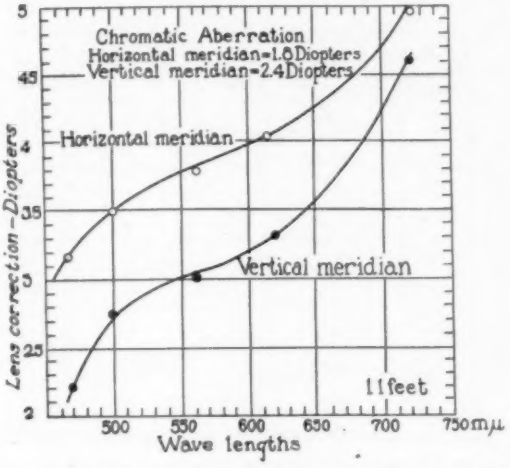

Fig. 11. *Chromatic aberration curves with fixation at 11 feet.*

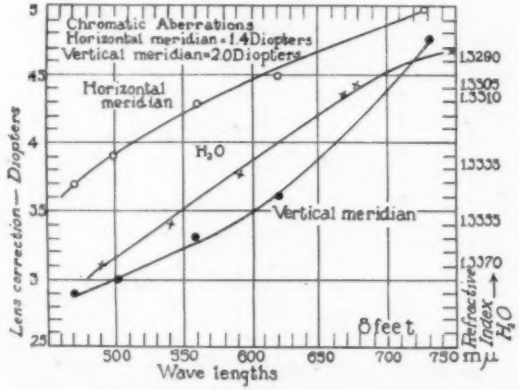

Fig. 12. *Chromatic aberration curves with fixation at 8 feet.*

although there are sets of data for fixation distances of 8, 5, and 3 ft., respectively, in which the chromatic aberration curve for the horizontal meridian is concave downward with reference to the abscissa (wave lengths), while the corresponding curve for the vertical meridian is

concave upward. The chromatic aberration curves, however, both for the vertical and horizontal meridians, when obtained with fixation distances of 16 to 11 ft., respectively, are of the form obtained by Ames and Proctor, while the curves given for fixation distances of 2 ft. and 1

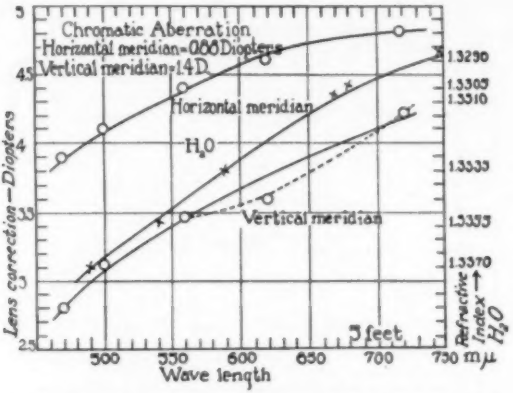

Fig. 13. *Chromatic aberration curves with fixation at 5 feet.*

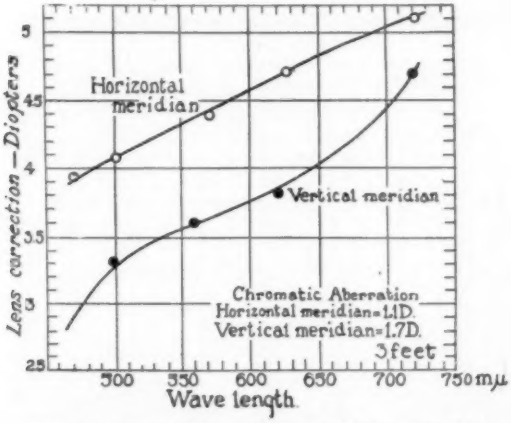

Fig. 14. *Chromatic aberration curves with fixation at 3 feet.*

ft., respectively, show the decided flattening in the 500 to 650 mμ region and are, therefore, in agreement with the results obtained by Nutting, indicating that, for my own eye, with fixation at short distances, there is more or less chromatic correction of the eye. Ames and Proctor suggested that the differences in forms of curves obtained by themselves

and by Nutting might be due to the latter's use of a white light as an accommodation object. I prefer to call this a fixation object, especially since light stimuli per se are very inferior accommodative stimuli as I shall presently show. Ames and Proctor used a monochromatic con-

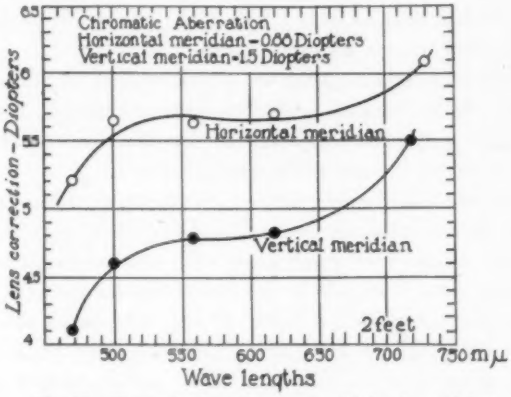

Fig. 15. *Chromatic aberration curves with fixation at 2 feet.*

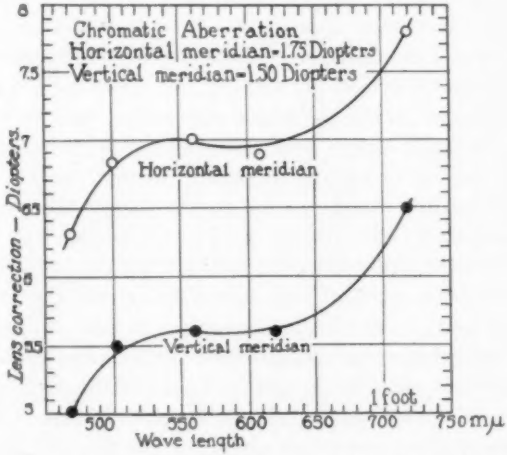

Fig. 16. *Chromatic aberration curves with fixation at 1 foot.*

trol. Their later experiments indicate that they obtained the same form of chromatic aberration curves with either white or monochromatic fixation; hence the question of a chromatic or monochromatic control (or fixation) does not appear to account for the difference.

The point may be raised as to whether accommodation was inactive or active in the investigations just referred to. From a description of the methods and other statements, one is led to believe that accommodation was inactive in the experiments of both investigators. From the data which I have obtained, it would be apparently immaterial whether or not the accommodation was active, since the determinations of refractive corrections for my own eye fixating points 1 ft., 2, or 3 ft. distant, at which distance accommodative action would be demanded for distinct vision under normal conditions such as reading, indicate quite definitely, that, under the conditions of the experiments, there existed little or no stimulus to accommodation (Table 2).

The size of the pupil might seem to supply a possible explanation. I therefore repeated observations at fixation distances of 22, 11, 5, and 3 ft., and 1 ft., respectively, using a disc with a circular aperture of 2 mm diameter. The data from which the chromatic aberration curves may be reconstructed are given in Table 1. Fig. 20 shows the chromatic aberration curves for the horizontal meridian only, with fixation at 22 and 5 ft., respectively. The shape of the curves in each set, that is, both with natural and diaphragmed pupils, is alike and in general the curves are parallel each to each. Furthermore, the value of the chromatic aberration has not been appreciably affected at either 22 or 5 ft. With fixation at 1 ft., however, as indicated by the curves in Fig. 21, there is a marked effect on the form of the curve and an appreciable effect on the value of the chromatic aberration, for the value of the chromatic aberration drops from 1.75 diopters with natural pupil to 1.3 diopters for the diaphragmed pupil. In fact, with a very small pupil at short fixation distances it appears as though the eye were somewhat overcorrected chromatically in the region from 550 to 700 mμ. This is further substantiated by the fact that the two chromatic aberration curves for the vertical and horizontal meridians, respectively, when plotted against fixation distance, cross at the 1 ft. fixation point (Fig. 19). I am led, therefore, to conclude that relative sizes of pupils of different observers, or those changes in the size of the pupil in any given eye which are the reflex result of the convergence induced by proximity of the fixation object, do exert some effect on the shape of the chromatic aberration wave length curves, especially when the pupils are very small and the fixation distance no more than 1 to 2 ft.

By means of suitable small apertures the peripheral portions of the refracting system of the eye may be eliminated. The use of a small diaphragm (2 mm aperture) discloses a greater refractive error in the eye

TABLE 1. *The effects of diaphragms on the refractive corrections, for various spectral regions and distances of the fixation source.*

Wave length	Meridian	Distance, feet	Without diaphragm, diopters	With diaphragm, diopters	Dioptric difference
4700	Horizontal (180°)	22	+3.50	+4.00	+0.50
		11	+3.50	+4.00	+0.50
		5	+4.00	+4.50	+0.50
		1	+6.25	+5.75	−0.50
	Vertical (90°)	22	+2.50	+3.00	+0.50
		11	+2.25	+2.87	+0.62
		5	+2.75	+3.50	+0.75
		1	+5.00	+4.50	−0.50
5600	Horizontal (180°)	22	+4.00	+4.75	+0.75
		11	+3.87	+4.50	+0.62
		5	+4.25	+5.25	+1.00
		1	+7.00	+6.75 or +7.00	−0.25 or 0.00
	Vertical (90°)	22	+3.12	+3.87	+0.75
		11	+3.00	+3.70	+0.70
		5	+3.50	+4.50	+1.00
		1	+5.50	5+.25	−0.25
6200	Horizontal (180°)	22	+4.10	+4.60	+0.50
		11	+4.00	+4.80	+0.80
		5	+4.60	+5.50	+0.90
		1	+7.00	+6.50	−0.50
	Vertical (90°)	22	+3.20	+4.00	+0.80
		11	+3.20	+4.00	+0.80
		5	+3.60	+4.70	+1.10
		1	+5.60	+5.00	−0.60
7300	Horizontal (180°)	22	+4.75	+5.12	+0.37
		11	+4.87	+5.50	+0.62
		5	+4.87	+5.50	+0.62
		1	+7.75	+7.00	−0.75
	Vertical (90°)	22	+4.20	+4.60	+0.40
		11	+4.60	+5.00	+0.40
		5	+4.30	+4.90	+0.60
		1	+6.50	+5.60	−0.90

than when the normal-sized natural pupil (5 to 8 mm) is used. This is as it should be, since the amount of hyperopia disclosed in most eyes, when tested by objective methods of skiascopy, is less in the peripheral

regions than in the central or so-called visual zone. In turn, the peripheral refraction is generally greater than that of the central zone in myopia. The elimination of the peripheral refractive zones does not, however, appreciably affect the shape of the chromatic aberration curve nor reduce the value of the chromatic aberration at fixation distances greater than 5 ft. (Figs. 20 and 21).

The effect of dilatation of pupil on the chromatic aberration, in contradistinction to and in comparison with the effects of small aperture diaphragms, is shown in Fig. 17. In this instance, the eye under examina-

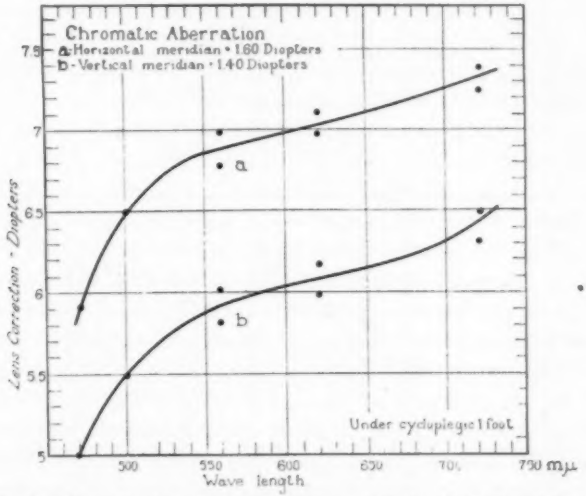

Fig. 17. *Chromatic aberration curves under a cycloplegic with fixation at 1 foot.*

tion was tested some few hours after the use of a cycloplegic. The results depicted in Fig. 17 should be compared with the forms of curves and the values of the chromatic aberration shown for 22 ft. (Fig. 9) and for 1 ft. (Fig. 16), in both of which the ocular pupil was of natural size. Since the eye was under a cycloplegic and since its amplitude of accommodation was found by suitable tests to be less than 1.5 diopters, and, furthermore, since there is, as will be shown later, a much reduced stimulus to accommodation at close fixation points by reason of the experimental methods pursued, it was to be expected that the general shape of the chromatic aberration curve obtained with fixation at 1 ft. under a cycloplegic would be similar to the curve of the eye with natural pupil passively fixating an object at 22 ft. There is, in other words,

little or no indication of achromatism with a pupil dilated under a cycloplegic fixating an object at a distance of 1 ft., whereas, without dilatation but rather with contraction of pupil and with no greater amount of accommodation involved in the one condition than in the other, there is evidence of considerable achromatism. These tests, therefore, further substantiate the suggestion that the size of the pupil does exert an effect on the shape of the chromatic aberration curves, the values of the chromatic aberrational intervals, and the degrees of achromatism exhibited.

One point of difference between the results shown in Figs. 17 and 9 seems worthy of mention. Under a cycloplegic (Fig. 17) the refractive difference between 470 and 560 mμ is practically 1 diopter for either the horizontal or the vertical meridian, while with natural sized pupil (Fig. 9) the refractive difference for this same spectral interval is 0.6 diopter. At the greater wave length end of the spectrum (620 mμ to 720 mμ), the refractive interval is 0.3 diopter (Fig. 17) and from 0.6 to 1.0 diopter (Fig. 9) for each principal meridian respectively. The total aberrational intervals are comparable in all cases, but the ranges of refractive corrections on the blue and red sides of the 620 mμ region are considerably different in the two instances. This observation was readily checked by viewing a distant street lamp at night, one eye being under the influence of a cycloplegic, the other in its natural condition, correcting lenses being worn. The color of the light, as well as of the surrounding corona, was decidedly reddish as seen by the eye under cycloplegic.

A comparison of the aberrational intervals in the eye under a cycloplegic fixating an object 1 ft. away with the same intervals in the natural eye fixating an object 22 ft. distant, on the one hand, and 1 ft. away on the other hand, shows that the values and the relative orders of the chromatic aberration for the two meridians of the eye under a cycloplegic quite closely approximate those for the eye with natural (hence small) pupil fixating the test-object 1 ft. distant. This lends support to the idea that the variation in incidence or obliquity of light entering the eye is to be considered in explaining these results, as well as the effects of size of pupil.

I believe, therefore, that the differences existing in the forms of shapes of chromatic aberration wave length curves obtained by various observers cannot be explained wholly by any one of the following: (1) character of accommodation object, as to whether it is white or monochromatic; (2) differences in accommodation by reason of the

distance of the fixation light stimulus, since the stimulus to accommodation is small in any event; (3) changes in size of pupil, except possibly in cases of small pupils with fixation at close distances (3 ft. or less); and (4) variations in spherical aberration and the elimination of certain refractive zones by suitable diaphragms.

I am unable to offer any explanation for these various results other than to suggest a composite effect due to: (1) spherical aberration; (2) change of size of the pupil under the stimulus of convergence; (3) anatomic variations in the structure of the crystalline lens so that the effective refractive indexes vary from zone to zone, or from the periphery inward; (4) areas or zones of the dioptric system of the eye, chiefly of the crystalline lens, which are operative in affording sharply focused images on the retina with minimal expenditure of energy on the part of the intrinsic muscles; and (5) the differences in obliquity or angles of incidence of the light entering the eye, dependent on the distance of the fixation object.

Since the refractive indexes for the optical media of the eye do not differ greatly from the index of water it might be expected that the curves for the axial focal lengths of the eye for different wave lengths of light would be similar to the refractive index wave length curve for water. Since $100/F \propto D$ and $1/F \propto (n-1)$—where F represents the focal length of a lens in centimeters, D the dioptric power and n the index of refraction—it follows that $D \propto (n-1.)$ That is, the dioptric power for any given wave length is proportional to the index of refraction of the medium for that wave length minus the constant, unity. Figs. 12 and 13 show a very close approximation of the chromatic curve of the eye to the refractive index curve of water. Even in these cases, however, the eye is overcorrected in the blue-violet region and undercorrected in the red.

Before closing this discussion on the forms of chromatic aberration curves, some comment should be made with regard to the significance of Fig. 18. The lowest curve of the three given, indicating the reversal points in the subjective shadow test, was obtained as follows. I placed before the eye under test an arbitrary correction, that is a correction which I often wear while engaged in reading and other close work, and determined the neutral or reversal points for various spectrally filtered fixation sources of light. The curve obtained shows that the correction worn, namely +5.25 D. S.+0.50 cyl. ax. 90, overcorrected the eye in the region from 450 to 600 mμ and undercorrected it in the region from 650 to 750 mμ and that this correction, with a fixation dis-

tance of 2ft., remained unchanged for the intervening region from 500 to 650 mμ, hence giving a definite flattening of the curve and indicating chromatic correction over a considerable portion of the visible spectrum. The upper curve of Fig. 18 gives the form of curve for the far-points of the eye tested when the above specified correction was worn. It is obtained by converting into diopters the distances of the ocular neutral point from the fixation object. It indicates that the correction selected overcorrects the eye in the visual region of lesser wave length by about 0.6 diopter and undercorrects in the region of the greater wave length by about the same amount. The second curve of Fig. 18 shows the amounts which would need to be added to the correction worn (+5.25 D. S.+0.50 cyl. ax. 90) in order to secure the proper chromatic

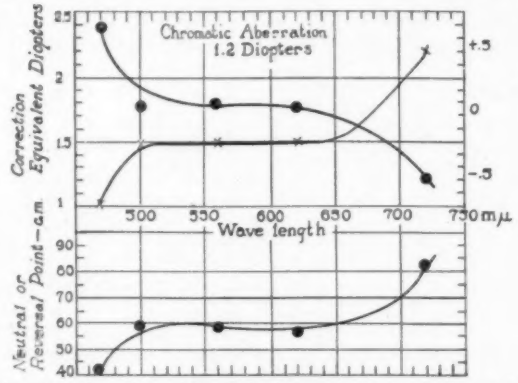

Fig. 18. *Chromatic aberration curves obtained through a determination of the neutral subjective skiametric distances, when an arbitrarily chosen refractive correction was worn.*

aberration curve. This curve should be compared with the horizontal meridian curve for 2 ft. fixation distance given in Fig. 15. It is to be noted that the forms of curves and numerical gradations (+5.75 D. S. in the horizontal meridian) are identical with those given in the central curve of Fig. 18. This method of finding reversal points when appropriate arbitrary corrections are used, if considered in the light of the refractive values determined at different fixation distances, affords an excellent check on the accuracy of these subjective skiascopic methods.

Fig. 19 gives a summary of the values of the chromatic aberration for various spectral regions of maximal intensity in the range of from 470 to 730 mμ and for the two principal meridians of refraction. It is of interest to note the maximal and minimal chromatic aberrational

intervals, ranging from 1.5 to 2.4 diopters in the vertical meridian and from 0.88 to 1.8 diopters in the horizontal meridian. The chromatic interval passes through the same relative changes of value in both meridians, as is evidenced by the parallelism of the two curves, until the fixation distance of 1 ft. is reached, at which point the curves cross and the value for the horizontal meridianal chromatic aberration becomes equal to or slightly greater than that in the vertical meridian.

It is to be noted that the chromatic aberration as measured in the vertical meridian of the eye is practically 0.5 diopter greater than in the horizontal meridian (Fig. 19). These experimental determinations,

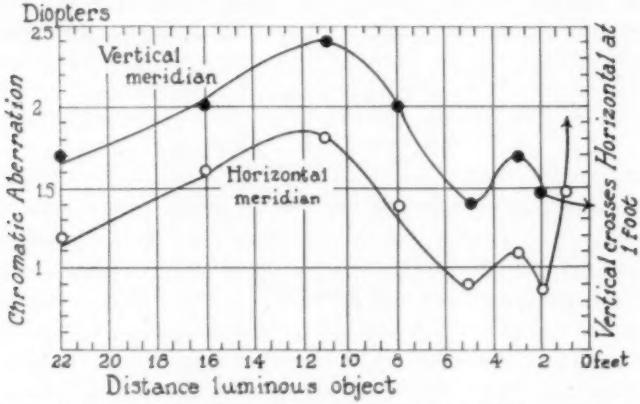

Fig. 19. *Composite curves for the chromatic aberrational intervals of the two principal refractive meridians of the eye.*

as made on an eye which has an astigmatism "with the rule," in which the correcting lenses would have incorporated a plus cylinder at or about 90° or a minus cylinder at or about 180°, may offer some explanation of the observation frequently made by refractionists that as the lenses used in subjective testing, when employing what is technically spoken of as the "fogging system" (the initial large overcorrection of hyperopia or undercorrection of myopia) are slowly reduced so that the visual acuity is not better than 4/10 to 6/10, an astigmatism "against the rule" evidences itself, whereas, under further reduction of trial case lenses, the true fundamental condition of low amount of astigmatism "with the rule" is disclosed. This means, for example, that in a refractive condition ultimately corrected by a +0.50 diopter cyl. ax 90 (or thereabouts) with too heavy a fogging and tests attempted on this low astigmatic error, no astigmatism would be evidenced.

There is no reason, a priori, why the relation of chromatic aberration to fixation distance should not be the same in both meridians. The inference is, however, that the refractive indexes or anatomic structures are not the same in all parts of the eye, especially in the vertical and horizontal meridians. Such differences are not known, from other evidence, to exist so far as I am aware. Since, however, these experimental methods probably afford a measure of the variations of the refractive indexes of various zones of the eye, particularly of the crystalline lens in situ, it is possible that there are here disclosed facts which could not be arrived at by any other method.

THE EFFECTS OF DIAPHRAGMS ON THE REFRACTIVE CORRECTIONS

Figs. 20 and 21 give a few of the curves obtained when the natural pupil of the eye is used as aperture and also when the determinations are made with a 2 mm circular aperture in a disc placed in the back

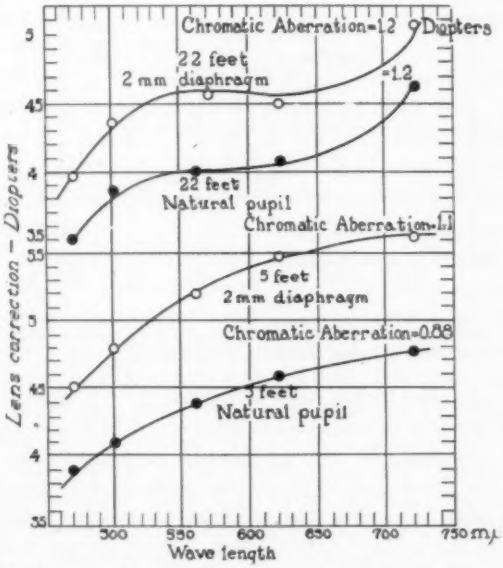

Fig. 20. *Diagram illustrating the effects of diaphragms on refractive findings.*

cell of a trial frame and in a position as close to the eye as is conveniently possible. Table 1 gives the data for various spectral regions and distances of fixation object.

The data and the curves suggest several interesting and important conclusions. First, the refractive corrections for fixation distances

from 22 to 3 ft. are greater with than without a diaphragm. This is in accord with findings as ordinarily made skiametrically with the mirror, for in most eyes the peripheral refraction is less hyperopic or more myopic than the central refraction. The order of this difference is from 0.5 to 1.0 diopter. A survey of Table 1 shows that such findings are corroborated by the subjective method of skiascopy. As a matter of fact, it is probably impossible to determine these variations with any objective methods as accurately as with the subjective method of skiascopy which has been presented and discussed. In a general way, the data show that the refractive difference between the natural and the diaphragmed pupil increases as the distance of fixation decreases until the distance (about 1 ft.) is reached, at which distance

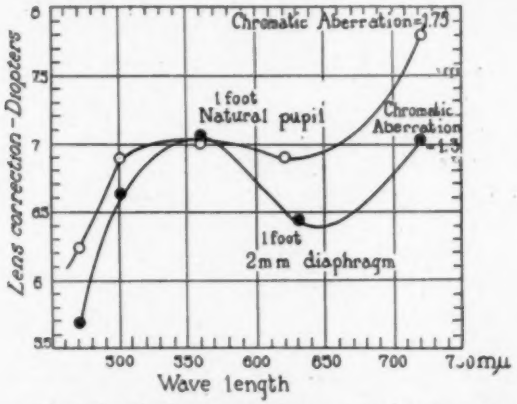

Fig. 21. *Diagram illustrating the effects of a diaphragm on the refractive findings and chromatic aberration curves obtained with fixation at 1 foot.*

the effects of the diaphragm as a substitute for or equivalent of accommodation become appreciably operative. These data are in conformity with observations and well-known facts as to the difference between the peripheral and central refraction of any plus or convex lens. In practical refraction, the presence of positive spherical aberration would entail a lower hyperopic or higher myopic correction in the peripheral regions.

The results given in Fig. 21 are not so readily explained, for the findings with the diaphragm are less than with the natural pupil. It might be urged that the actual pupil, by reason of the proximity of fixation object (the distance being 1 ft.) had become smaller than the aperture of 2 mm diameter, by reason of a combination of con-

vergence, accommodation and reaction to light. From auxiliary determinations I judge that this was not the case, although there is normally a very considerable diminution in pupillary size. With short fixation distances, measurements showed that the apparent diameter of the eye under test was between 3.5 and 4 mm. In the regions of the visual wave lengths, the findings with and without diaphragm approach each other in value and become experimentally equal at 560 mμ, which is practically the region of maximal sensibility of the eye for low illumination. In the region of greater wave lengths, however, there is a reduction in the power of the correcting lens of from 0.75 to 1.0 diopter when a small diaphragm is used. The reversal of the relation of the diaphragmed to the undiaphragmed pupil may be attributed to either one or both of the following: the increased acuity or the equivalence of correcting lens power produced by small apertures, and the increased accommodative action due to the stimuli of the longer light waves. The second possibility is taken up and discussed in greater detail elsewhere in this paper.

Fig. 21 gives experimental proof of the important part which very small pupils play in enabling presbyopes, often of advanced years, to read without any correcting lenses or else with lenses which incorporate distance corrections only. In such cases it has generally been presumed that one of three conditions existed: (1) amplitudes of accommodation were considerably in excess of those ordinarily possessed by persons well advanced in years; (2) myopia or myopic astigmatism was present and had probably existed for years, and (3) certain pathologic changes, incident to incipient cataract or so-called "second sight," were present. As a matter of fact, the usual amplitude of accommodation of a person forty years of age is about 3.5 diopters and diminishes until, at about seventy years of age, the accommodative amplitude has decreased to zero. Ordinarily only one-half of the amplitude can be used continuously and with comfort. I have investigated the refractive conditions of a few persons, aged sixty years or more, who read the newspaper at ordinary reading distances, without spectacle lenses of any description. In these instances the static findings have shown small amounts of hyperopia. The amplitudes of accommodation, as determined by my objective test for amplitude of accommodation, have not been different from those which would be expected for the ages given. Hence, excessive accommodative amplitude cannot be given as the reason for this ability to read and do other close work, without auxiliary assisting lenses, both with comfort and for extended periods of time.

Such persons, however, invariably possess extremely small pupils. In the light of the data obtained (shown in part in Fig. 21) I believe that reduced pupillary apertures (of the order of 2.5 mm or less in diameter) act as the partial equivalents of accommodation. Fig. 21, for instance, shows that from 0.5 to 1.0 diopter less refractive correction is needed with a fixation distance of 1 ft. when an aperture of 2 mm is used than when the natural sized pupil is used.

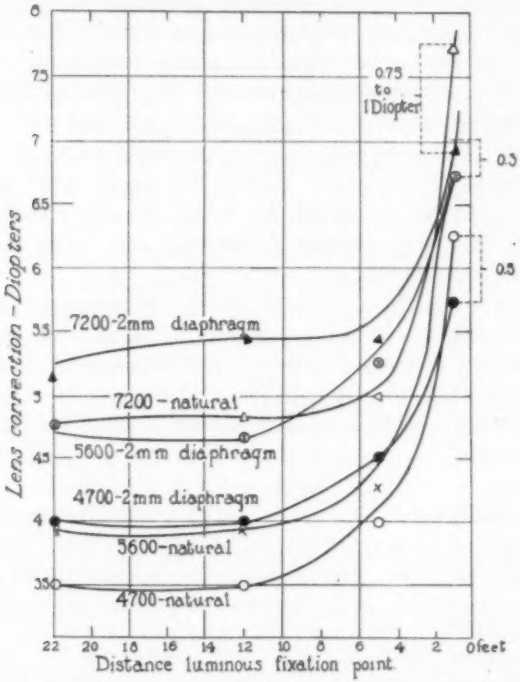

Fig. 22. *Diagram illustrating the effects of diaphragms on the values of the lens corrections for various wave lengths of the source of fixation.*

In recent years I[13,14,15] have advanced the theory that the refractive findings obtained by dynamic skiametric procedures at the customary

[13] Sheard, Charles: Dynamic skiametry and methods of testing the accommodation and convergence of the eye; being an essay contributed to the American Encyclopedia of Ophthalmology. Chicago, Cleveland Press, 108 pp.; 1920.

[14] Sheard, Charles: Ocular accommodation. Department of Education of American Optical Association, pp. 14-21; 1920.

[15] Sheard, Charles: The comparative value of various methods and practices in skiametry. Am. Jour. Physiol. Optics, *3*, pp. 177-208; 1922.

reading distance of about a foot will normally exceed the static skiametric findings by from 0.5 to 1.0 diopter of convex lens power. In order to explain this fact, which has been sufficiently substantiated by innumerable tests by numerous practitioners, I have suggested that there is normally a lag of the accommodation behind the convergence to an amount varying from 0.5 to 1.0 diopter. There are many experimental facts to substantiate this premise, but it would be out of place to discuss them here. It seems possible, however, in the light of the results which have been obtained with the use of diaphragms, to explain the normal excessive dynamic skiametric finding in hyperopia and decreased value in myopia, as compared to the static determination, by the hypothesis that reduction of pupillary aperture acts as the partial equivalent of accommodative action.

Fig. 22 shows the results (charted in Table 1) obtained with natural and diaphragmed pupils for the spectral regions indicated and for the fixation distances specified. The curve for 560 mμ with natural pupil is practically coincident with that for 470 mμ with a 2 mm aperture, and the other curves show similar relations. An inspection of the curves of Fig. 22, as well as the data contained in Table 1, show that, for any specified wave length chosen, the curve of refractive correction with natural pupil approaches the curve of refractive correction with 2mm diaphragm as the fixation distance is reduced, until a fixation distance of between 2 and 3 ft. is reached. At this distance the two curves for any given spectral region cross, or, in other words, the refractive corrections for both natural and diaphragmed pupil become equal. At a distance of 1 ft., which is about the normal reading distance, the refractive correction for the diaphragmed eye is less than that for the same eye with natural pupils by amounts varying from 0.3 to 1.0 diopter.

THE CHROMATIC ABERRATION IN THE INTERVAL OF STURM AND THE CHANGES IN THE AMOUNT OF ASTIGMATISM WITH VARIED FIXATION DISTANCES

There are several points of interest with reference to the data concerning the amounts of astigmatism exhibited at both distant and near fixation points and concerning the chromatic aberration in the so-called interval of Sturm, or the ocular astigmatic interval. The salient features of these experimental results are shown in Fig. 23. The curves showing the amounts of astigmatism obtained for 22, 11, 8, 5, 3 and 2 ft., respectively (some of which are reproduced in the figure), are essentially the same in character and in the amount of chromatic aberration,

which is of the order of 0.5 diopter. Each curve was determined by taking the difference of the refractive corrections in the two principal meridians of the eye for any specified spectral fixation region and distance. Contrary to the variations exhibited by the chromatic aberration curves for the principal meridians with various fixation distances, these curves show that the amount of astigmatic error is practically constant at 0.87 diopter throughout the region from 500 to 650 mμ.

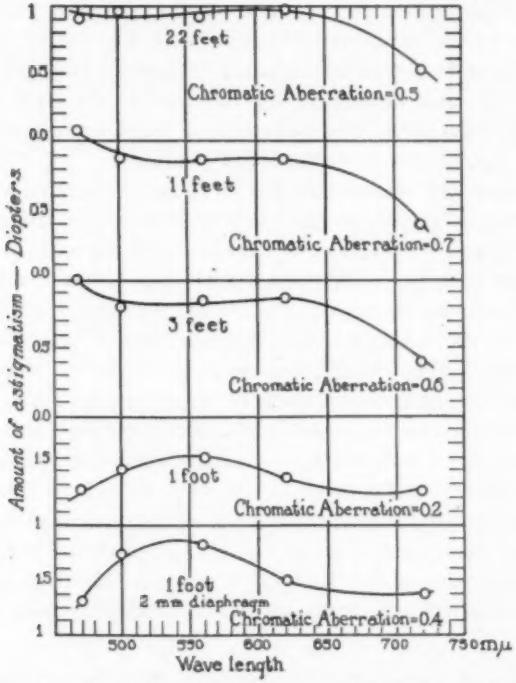

Fig. 23. *Diagram illustrating the variations in the amounts of astigmatism and forms of chromatic aberrational curves in the interval of Sturm, under various distances of fixation.*

and, therefore, that the eye is quite completely or, at least, very considerably corrected for the major portion of the visual spectrum, with respect to any differences in refractive values for the two principal meridians, through the correction of its astigmatic error. Since this is experimentally true for both of my own eyes and two others examined, the conclusion may be ventured, subject to further test, that the criterion for the most perfect vision possible, from the chromatic standpoint, is that there shall be a minimal difference in the corrective

values or a maximal achromatism in the two principal meridians of the eye in those portions of the spectrum (500 to 650 mμ) in which the eye is most efficient. It is far more desirable optically that all portions of an object under regard should be focused, both as to form and color, in the same plane, than it is that all objects, of whatever color and at whatever fixation distance specified, be focused in an identical plane. There is, therefore, a significant reason for the presence of considerable achromatism in the interval of Sturm. Since nearly all eyes possess a small amount of astigmatism, it would appear as though this criterion of achromatism in the astigmatic interval were universal. It would also be highly desirable that such a condition should exist at the reading distance; otherwise all parts of an object viewed in a given plane, or flat surface, at a short fixation distance would never be distinctly seen, at least simultaneously. It is possible, also, that these experimental results give additional reasons why it is so taxing to read under red or blue lights. These data also explain why it is that, when subjective tests are made on astigmatism, by means of the fan test or clock-dial, those under test frequently report that one set of lines appears black and very distinct, whereas those at right angles are much less distinct but fringed with red.

The curves for 22, 11, and 3 ft. fixation distances (Fig. 23) show that the astigmatic interval would be slightly undercorrected in the region from 400 to 500 mμ and overcorrected (about 0.5 diopter) in the region from 650 to 750 if it were to be corrected by a cylindrical lens the value of which is that given by determinations made in the region from 500 to 620 mμ. It is to be noted that the undercorrections for the regions of short wave lengths and the overcorrections for those of long wave length in the astigmatic interval are the reverse of the conditions which would hold if the refractive corrections called for in the region from 560 to 620 mμ, as shown in the chromatic aberration wave length curves of Figs. 9 to 16, were to be given for either one of the two principal meridians.

At such a short fixation distance as 1 ft., however, a condition is disclosed in which the correction of the astigmatic interval shows practically no differences in the amounts demanded in the various spectral regions. If anything, the indicated correction of 1.5 diopter as determined for the region from 550 to 620 mμ would slightly overcorrect both ends of the visible spectrum.

A comparison of the last two curves of Fig. 23 shows that the use of a 2 mm diaphragm did not change the form of the astigmatic wave

length curve, although in this particular instance the values of the astigmatic error are slightly higher when the diaphragm was used. A reference to the data in Table 1 shows that, in general, the forms of the curves and the values of the astigmatic corrections did not change with the use of diaphragms in cases of fixation distances ranging from 22 to 2 ft.

The curves in Fig. 23 also lend support to some recent but as yet unpublished investigations by King and Sheard which show that the most sensitive subjective tests for astigmatism, which can be readily used in the practice of refraction, are afforded by a suitable glass filter placed before the eye under test and giving a high transmission in the yellow-orange region only. This selective glass filter is used in conjunction with a chart whose background carries a color corresponding to the region of maximal transmission of the glass filter, the radiating lines on the chart being black.

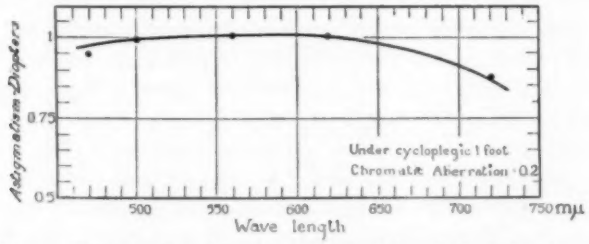

Fig. 24. *Diagram illustrating the astigmatic wave length relations obtained when the eye was under a cycloplegic and fixating the test object at a distance of 1 foot.*

The question has often been discussed whether or not the amount of astigmatism in an eye possessing this defect remained at a constant value when the eye was engaged in distant vision, on the one hand, and in close work such as reading, on the other hand; and whether or not such a condition as accommodative astigmatism exists. Various opinions have been held. Anatomically it is possible for the ciliary muscle to contract or relax to a greater degree in one meridian than in another. The results on which Fig. 23 is based indicate that the value of the astigmatism remains constant down to a distance of from 1 to 1.5 ft., and that at a fixation distance of 1 ft. its amount is appreciably increased. For fixation distances of from 22 to 2 ft. the amount of astigmatism in my own eye, remains constant at 0.87 diopter, but at 1 ft. it has a value of 1.3 to 1.5 diopters, or an increase of approximately 0.62 diopter.

The astigmatic wave length curve, shown in Fig. 24, was obtained with the eye under a cycloplegic and while fixating the spectral fixation object at a distance of 1 ft. A condition of achromatism in the region from 500 to 620 mμ is again exhibited. The value of the astigmatic correction under a cycloplegic is shown to be 1 diopter, with which value the finding of 0.87 diopter without a cycloplegic is in excellent agreement. A comparison of the curve of Fig. 24, taken under a cycloplegic, with the curve of Fig. 23, obtained without the use of a cycloplegic but with the same fixation distance of 1 ft., clearly proves that accommodative astigmatism may exist, and the amount of astigmatism disclosed, with or without a cycloplegic, under distant fixation or conditions of static testing, may be exceeded when the eye is dynamically functioning.

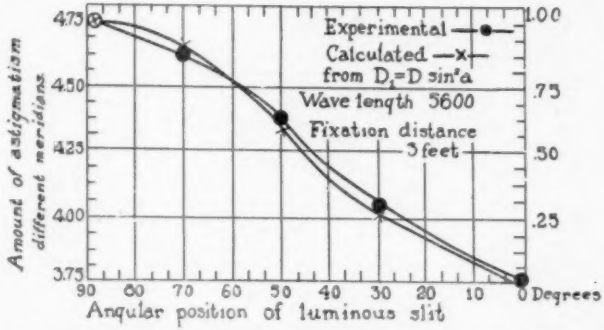

Fig. 25. *Diagram illustrating the theoretical and experimental values of the astigmatic corrections obtained with changes in the angular position of the luminous slit.*

Fig. 25 gives the values of the refractive findings in the various angular meridians of test indicated by the abscissae, when the fixation distance was 3 ft. and the spectrum filter giving maximal transmission at 560 mμ was used. The slit was set at the various angles indicated and the testing object was moved parallel to the direction of the slit. This parallelism of the slit and the moving wire or straight-edge that serves as a test object can be readily determined by simple observation. The line with the circles (●) gives the experimental determinations, while the line with crosses (×) gives the curve calculated from the relation

$$D_1 = D \sin^2 a$$

in which D represents the maximal astigmatic correction (practically 1 diopter in this instance), and D_1 indicates the value of the astig-

matic error at the angle (α) specified. Several such curves for different fixation distances and selected spectrum filters were taken and in practically every case the experimental curve exhibited values slightly greater than those given by calculation. Moreover, such curves as these give one an appreciation of the sensitiveness of the subjective method of skiametry, although it requires some patience and keenness of observation to recognize differences as close as 0.12 diopter with a possible limit of 0.06 diopter.

THE ABSENCE OF STIMULUS TO ACCOMMODATION WHEN FIXATION OBJECTS OF SPECTRALLY FILTERED LIGHT ARE USED

Possibly one of the most important results and conclusions from these investigations on the subjective methods of skiascopy and its applications to the study of the aberrational characteristics of the eye lies in the statement that light and color, per se, as obtained by the use of filtered spectral regions for fixation objects, supply but little, if any, stimulus to accommodative action.

In order to present all the data from which certain conclusions will be drawn, it is necessary to rehearse a considerable number of auxiliary facts. The curves of Fig. 9 show that the static refraction, with fixation at 20 ft. or more, as obtained by the subjective skiametric method, at wave lengths 550 to 620 mμ, is +3.5 D. S.+0.75 cyl. ax. 90. As has been pointed out, however, this finding is in reality a measure of the peripheral refractive error, whereas the central refraction, measured in the so-called visual zone, or with the use of a diaphragm. (Fig 20), is +4.00 D. S.+0.75 cyl. ax. 90. Anticipating somewhat certain conclusions which are to be drawn, it seems desirable at this place to point out the fact that the peripheral refraction is more vital to distinct distant vision than is the central refraction. That is to say, the peripheral zones play a more prominent part in the distinct vision of distant objects, while the central zones (since the peripheral zones are quite largely eliminated by reason of pupillary contracture) function as the sole areas of refraction in close vision. Also, at short fixation distances, accommodation is normally involved in the act of distinct vision and is normally supplied unless certain uncorrected refractive conditions or presbyopia prohibit its functioning.

Under a cycloplegic, the objective static skiascopic determinations and subjective findings gave +4.00 D. S. +0.75 cyl. ax. 90. This finding is the same as that of the refraction in the central zone obtained by the subjective skiascopic methods, and clearly indicates that there

is a tonic condition of accommodation of about 0.5 diopter which is relaxed under cycloplegics. It is quite customary, when using cycloplegics in refractive work, to deduct a dioptric value varying from 0.5 diopter to 1.0 diopter, from the findings in hyperopic conditions, and to add a like amount in myopic conditions as a basis for subsequent tests without cycloplegics. The reason is quite obvious from these experiments. Moreover, it appears that these and similar results lend support to the Helmholtz theory of accommodation in that, with relaxation of the ciliary muscle, the central and neighboring peripheral refractions become practically the same.

Fig. 17 shows the refractive corrections for various spectral fixation sources with the eye under a cycloplegic and fixating an object 1 ft. distant. For the region 630 mμ the findings in the two principal meridians are practically 7 and 6.25 diopters respectively. Under a cycloplegic an eye possesses little or no accommodation, depending somewhat on the drug used. In the case of the eye under test, an examination of the accommodative status of the eye, when equipped with +4.00 D. S.+0.75 cyl. ax. 90, and a few hours after the instillation of the cycloplegic, showed that there was a range of accommodation extending from 24 to 39 cm, indicating an amplitude of accommodation of 1.5 diopters. The same eye, without a cycloplegic and wearing the same correcting lens, showed an amplitude of accommodation of 4 diopters. From a comparison of the results shown in Fig. 17 with those given for the static refraction at 22 ft. (Fig. 9), with reference again to the findings at 620 mμ, one of two conclusions is evidently correct: (1) that, allowing 3 diopters as the optical equivalent of 1 ft., and presuming that no accommodation takes place while fixating, under a cycloplegic, an object 1 ft. away, the findings are 4 and 3.25 diopters, respectively, for the two principal meridians of refraction, hence giving a refractive correction at 22 ft. corresponding to the peripheral refraction obtained by the static subjective skiascopic method, or (2) that, assuming the finding of +4.00 D. S.+0.75 cyl. ax. 90 under a cycloplegic to be correct, there is an accommodation of about 0.5 diopter demanded and supplied in the eye under a cycloplegic while fixating an object 1 ft. away. I believe that the second premise is the more probable one. At any rate, irrespective of the foregoing deductions, comparisons of the values obtained by the subjective skiascopic method with those under a cycloplegic clearly demonstrate that little or no accommodation is involved at any fixation distance in these subjective skiascopic procedures.

Figs. 26 and 27 and Table 2 summarize the lens corrections for various spectral regions and distances of the source of fixation. Fig. 26 gives the data for the horizontal meridian and Fig. 27 for the vertical meridian. Table 2 contains data for fixation distances of 3, 2, and 1 ft., respectively, together with such auxiliary data as are necessary in order to permit of the determination of the amount of accommodation actually supplied under any of the conditions specified. An inspection

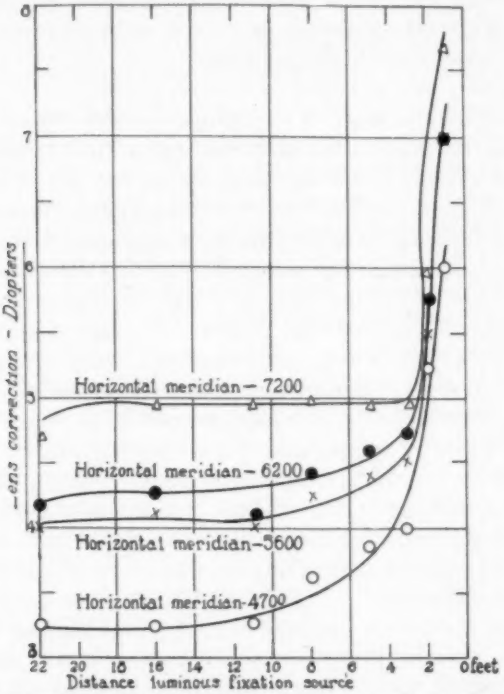
Lens correction - Diopters
Horizontal meridian— 7200
Horizontal meridian— 6200
Horizontal meridian—5600
Horizontal meridian-4700
22 20 18 16 14 12 10 8 6 4 2 0 feet
Distance luminous fixation source

Fig. 26. *Graphical summary showing the relations between the refractive corrections and the spectral radiation of the source of fixation.*

of Table 2 and of the curves of Figs. 26 and 27, shows that: (1) the accommodation involved at any fixation point within the range of 22 ft. and 1 ft. (hence practically from infinity to the reading distance) is but a fraction of that normally demanded; (2) the stimulus to accommodation is greater for the longer visual wave lengths (600 to 750 mμ) than for the shorter wave lengths, and (3) the subjective skiascopic method affords a procedure in refractive work whereby

the refraction can be determined under a natural and practically complete inhibition of accommodation, and with the pupils of natural

TABLE 2. *The relations between the refractive corrections, regions of spectral radiation and accommodative stimuli.*

Wave length	Meridian	Distance of test, feet	Correction for distance of test, diopters	Correction for 22 feet, diopters	Difference of correction for 22 feet and point specified, diopters	Amount of accommodation normally exerted, diopters	Amount of accommodation experimentally determined, diopters
4700	Horizontal (180°)	3	+4.00	+3.25	+0.75	1.00	0.25
		2	+5.25		+2.00	1.70	0.30
		1	+6.25		+3.00	3.00	None
	Vertical (90°)	3	+3.00	+2.50	+0.50	1.00	0.50
		2	+4.00		+1.50	1.70	0.20
		1	+5.00		+2.50	3.00	0.50
5600	Horizontal (180°)	3	+4.50	+4.00	+0.50	1.00	0.50
		2	+5.50		+1.50	1.70	0.20
		1	+7.00		+3.00	3.00	None
	Vertical (90°)	3	+3.60	+3.00	+0.60	1.00	0.40
		2	+4.75		+1.75	1.70	None
		1	+5.50		+2.50	3.00	0.50
6200	Horizontal (180°)	3	+4.75	+4.25	+0.50	1.00	0.50
		2	+5.75		+1.50	1.70	0.20
		1	+7.00		+2.75	3.00	0.25
	Vertical (90°)	3	+4.00	+3.25	+0.75	1.00	0.25
		2	+4.75		+1.50	1.70	0.20
		1	+5.50		+2.25	3.00	0.75
7200	Horizontal (180°)	3	+5.00	+4.75	+0.25	1.00	0.75
		2	+6.00		+1.25	1.70	0.45
		1	+7.75		+3.00	3.00	None
	Vertical (90°)	3	+4.75	+4.50	+0.25	1.00	0.75
		2	+5.50		+1.00	1.70	0.70
		1	+6.50		+2.00	3.00	1.00

size and functioning in a normal manner, contracting under the influence of the impulse of convergence as the spectral fixation object is approached toward the eye.

It has been possible for many years to produce, by the use of proper drugs, either dilatation of pupil without inhibition or paresis of accommodation, or inhibition or paresis of accommodation with dilatation of pupil. Accommodative inhibition has always been a necessary adjunct to accurate findings in refraction but the dilatation of pupils, from the refractive standpoint, has been a complicating factor and one which every refractionist who uses cycloplegics as a part of his refractive routine might well desire to have eliminated. The subjective skiascopic

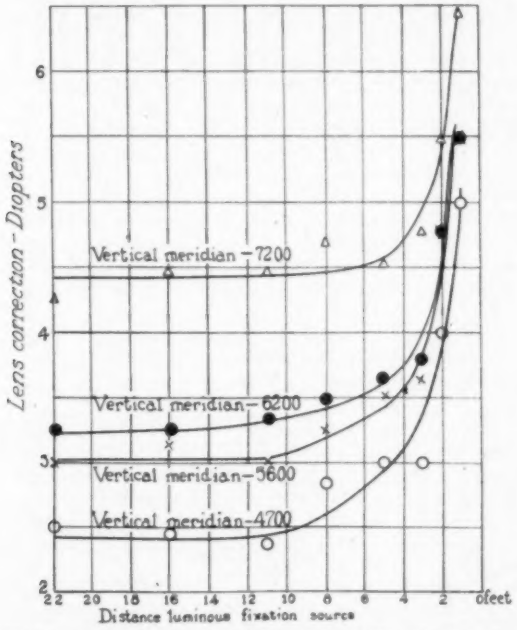

Fig. 27. *Graphical summary showing the relations between the refractive corrections and the spectral radiation of the source of fixation.*

method furnishes the desideratum, namely the inhibition of accommodation, with pupillary apertures changing in size as the natural accompaniment of the act of convergence. The unsatisfactory feature of the method to those engaged in practical refraction, however, at least in its present state of development, lies in the difficulty of making it a fairly rapid procedure.

It is to be hoped that the subjective method of skiascopy will find further applications to visual problems and open up other fields of

research in physiologic optics. I propose to apply this method further to a comparison of refractive data obtained by tests of visual acuity and by subjective skiascopy, and a comparison of findings by various objective and subjective skiametric methods.

SUMMARY AND CONCLUSIONS

Subjective skiascopy

This paper deals with the theory and practice of subjective skiascopy (retinoscopy, or skiametry) together with deductions drawn from the applications of this method to various points in physiologic optics.

In the ordinary methods of skiascopy as practiced objectively the operator determines the refractive condition of an eye under test by observation of the movements of the so-called retinal reflexes and shadows as viewed in the observed pupil. In subjective skiascopy, the person under examination regards the spatial images of the shadows cast on the retina by means of a suitable test-object moving between the eye and a fixation source of light. The directions of movement of test-object and spatial image (shadow seen on the fixation source of light) in subjective skiascopy are the reverse of those obtained by an examiner when objective skiametric methods and a plano-mirror are used.

Details are given regarding: (1) suitable luminous fixation objects, screens and diaphragms, (2) backgrounds, and (3) forms of the test-object producing the retinal shadow.

Under repeated test, the subjective skiascopic method is capable of an accuracy of 0.12 diopter. Its accuracy and its freedom from dependence on Snellen test letters and other devices for testing visual acuity commend it as an excellent test to add to the refractionist's armamentarium; its lack of rapidity of execution and its dependence on keen discrimination as to the presence or absence of shadow movement are serious hindrances.

The subjective method of skiascopy has been used to investigate or determine the following:

1. The amounts of refractive error, or correction needed, for fixation distances ranging from 22 to 1 ft.

2. A comparison of the values of the peripheral refraction with those obtained in the central area or so-called visual zone.

3. The importance of a consideration of the peripheral findings in the determination of refractive findings for distinct distant vision, and of the central (visual zone) refraction in close vision when accommodation and contracted pupils are normally present.

4. The general effects of diaphragms, which are the artificial equivalents of naturally contracted pupils, on vision and the refractive corrections demanded.

5. The changes in both the amount of astigmatism and the characters of the astigmatism—wave length curves with change in fixation distance, especially at distances approximating 1 ft.

6. The variations in the astigmatic error in passing, by specified angular intervals, from the primary to the secondary refractive meridian.

7. The possibility of accommodative astigmatism, demonstrated by the fact that the astigmatic corrections at short fixation distances exceed those at 20 or more feet by from 0.5 to 0.75 diopter.

8. The depth of focus and the lag of accommodation under conditions in which there is present a minimal stimulus to accommodation. Both of these may be explained on the hypothesis that reduction of pupillary aperture acts as the partial equivalent of accommodative action.

9. The similarity of the subjective skiascopic findings, in every instance and for all fixation distances, and the refractive findings obtained under a cycloplegic, indicating that the subjective skiascopic method furnishes the theoretically ideal system in that there is inhibition of accommodation, with pupillary apertures changing in size as the natural accompaniment of the impulse to convergence.

10. Data were checked through the use of any arbitrary but suitable correction placed before the eye, and finding the distances at which the retina of the eye was conjugated respectively for the various luminous fixation objects and spectral regions employed.

Chromatic aberration and allied phenomena

The methods of subjective skiascopy have been applied to investigations on the chromatic aberration of the author's eyes, a study of the chromatic variations in the interval of Sturm, and a determination of the accommodative status of the eye fixating various spectral fixation objects placed at distant and near points. Wratten filters and other light combinations were used in connection with the accessory apparatus devised for these experiments. The experimental data cover five spectral regions with maximal wave lengths of 470, 500, 560, 620, and 720 $m\mu$ respectively. Since the author has an astigmatic error with the principal meridians vertical and horizontal, refractive corrections for both meridians in these five spectral regions were obtained for distances of luminous fixation source ranging from 22 to 1 ft.

The principal conclusions relative to chromatic aberration and allied phenomena are:

1. Some of the chromatic aberration curves presented are in agreement with the results of Nutting, showing a flattening of the curve or a condition of achromatism in the region from 550 to 650 mμ while other curves are in agreement with those obtained by Ames and Proctor, in which little if any of this flattening or indicated achromatism occurred.

2. These differences in the forms and shapes of the chromatic aberration curves, varying with the distance of the luminous fixation object from the eye and with the meridian investigated, are probably due to several causes, chief of which may be mentioned variations in the size of ocular pupil, in the structure of the eye in various zones, and in the degrees of obliquity of light entering the eye.

3. The forms of the chromatic aberration curves for the two principal refractive meridians taken at any given fixation distance are similar in certain instances and quite dissimilar in others. For one or two fixation distances the chromatic aberration curve for the horizontal meridian is concave downward while that for the vertical meridian is concave upward.

4. Under certain conditions chromatic aberration curves were obtained which correspond to the refractive index wave length curve for water.

5. For all fixation distances down to 1 ft. the values of the chromatic aberrational interval of the eye (from 470 to 720 mμ) for the vertical meridian exceed those obtained for the horizontal meridian by about 0.5 diopter. At a distance of 1 ft., these values become equal or reversed in the two meridians.

6. Composite curves showing the relation of chromatic aberration to fixation distance for both principal refractive meridians of the eye show successive maximal and minimal values of the chromatic aberration as the distance of fixation object is changed.

7. The values of the chromatic aberration for the interval from 470 to 720 mμ range from 1.5 to 2.5 diopters for the vertical meridian and from 0.9 to 1.75 diopters for the horizontal meridian.

8. There is considerable evidence, therefore, as deduced from these investigations, to support the statement that there is an approximate achromatism of the eye in the region from 540 to 620 mμ under certain conditions of size of pupil and fixation distance.

9. Astigmatism—wave length curves show that the astigmatic correction is greatest for 470 mμ, of equal but slightly lower value at 500, 560, and 620 mμ, and least in the region 720 mμ. The chromatic interval from 470 to 720 mμ in the invertal of Sturm ranges from 0.5 to 0.7 diopter for fixation distances down to the customary reading distance.

10. The variation in the amount of astigmatism with wave length becomes practically nil at 1 ft., indicating a condition of achromatism in the interval of Sturm at the customary reading distance. Small diaphragms used in conjunction with tests made at a fixation of 1 ft. support these findings and also show that the finding for 560 mμ overcorrects the regions of both the short and long wave lengths of the visible spectrum.

11. The conclusion is ventured that the criterion for the most perfect vision, from the chromatic standpoint, is that there shall be a minimal difference in the corrective values or, in other words, a maximal achromatism in the interval of Sturm for those portions of the spectrum (500 to 650 mμ) in which the eye is most efficient.

12. These investigations demonstrate that light and color, per se, as obtained by the use of selective spectral fixation objects, supply very little stimulus to accommodative action.

13. The longer wave lengths of the visual spectrum afford a greater stimulus to accommodation than do the shorter wave lengths.

SECTION ON PHYSICS,
MAYO CLINIC AND THE MAYO FOUNDATION,
ROCHESTER, MINNESOTA.

Magnetic Properties of Iron-Nickel Alloys.—Alloys having more than 34.4% of Ni are typically paramagnetic beyond the Curie-point (up to about 1200°C anyway) and the susceptibility S is given for each by an equation $S(T-\Theta)=C$, in which Θ is a constant slightly but systematically greater than the Curie-point temperature. Alloys having a greater iron-content are typically paramagnetic above a critical temperature, while between this and the Curie-point they display two anomalies: (a) non-linear relation between magnetizing field and magnetization, and (b) "a slow and prolonged evolution of the magnetization after temperature-equilibrium is attained." As the critical temperature in question "seems to" approach the transformation temperature between Feβ and Feγ as the Ni content of the alloy approaches zero, Peschard thinks that the said anomalies indicate a state of the iron-rich alloys corresponding to the β state of Fe. [M. Peschard; C. R., *181*, pp. 99-101; 1925.]

KARL K. DARROW

INSTRUMENT SECTION

THE ADAPTATION OF THE PHONELESCOPE AS A PRECISION PITCH INDICATOR, AND AN APPLICATION TO VOCAL TONES

By L. E. Dodd

INTRODUCTION

An investigation[1] of the pitch effect of external pressure abruptly applied to a singer's throat while the subject is sounding a vocal tone, required some sensitive experimental device to detect pitch change. In considering mechanical instruments having sensitive response to sound waves, Miller's phonodeik[2] comes at once to mind as probably the most sensitive device of the kind that has yet been developed.[3] It was designed, primarily at any rate, for the precision analytical study of tone wave forms. It might be adapted as a very sensitive pitch indicator, by permitting the vibrating beam to fall upon a suitable stroboscopic screen. We should thus have combined a highly sensitive mechanical device with the extremely precise stroboscopic method. Such an adaptation would afford direct observation of both the pitch and any pitch change of a sounding tone. However, as a stroboscopic pitch indicator of this type, the phonelescope[4] which bears considerable similarity to the phonodeik, is perhaps as precise, and is sufficiently sensitive to give stroboscopic response with tones of relatively low intensity. Moreover, it is an instrument readily available. It was therefore chosen for the investigation mentioned.

APPARATUS

A schematic diagram of the assembled apparatus used for the present purpose is shown in Fig. 1, which gives the approximate relative locations of the parts as seen from above. *A* is an arc source (a pointolite is advantageous for steadiness), light from which is converged upon a small aperture, *O*, then diverges to the entrance aperture of the phonelescope, *P*, emerging at the lens window of this instrument, which forms

[1] Reported at the Pasadena meeting of the American Physical Society, March, 1925.

[2] D. C. Miller, The Science of Musical Sounds, Macmillan, 1922.

[3] S. H. Anderson has recently modified the phonodeik. See J.O.S.A. & R.S.I., *2*, p. 31, 1925.

[4] H. G. Dorsey, J.O.S.A. & R.S.I., 6, p. 279, 1922.

a real image of the aperture O on the stroboscopic screen, S. Two mirrors, M_1 and M_2, aid in directing the beam so that the subject can observe the pitch as he sounds the tone into the receiving horn, H. As receiving horn the recording horn of a phonograph was attached to the phonelescope by a short length of rubber hose. Only the axial ray of the converging cone travelling from P to M_1 to M_2 to S is shown. The vibrational character of the beam is indicated.

The mechanism of the phonelescope itself is essentially simple. It will not be described here, except to say that a diaphragm vibrates in response to the sounded tone, and causes a mirror attached to it by a

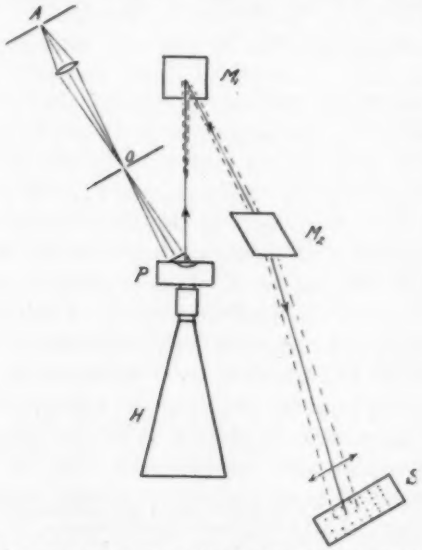

FIG. 1. *Schematic diagram showing adaptation of phonelescope to produce stroboscopic effect.*

suitable communicating system to vibrate in a similar manner. The vibrating mirror in turn produces corresponding movements in a beam of light reflected from it upon a screen.

For the stroboscopic screen, a hollow brass cylinder, or drum, having an outside diameter of 56.4 mm, approximately equal in size and similar in appearance to an Edison phonograph cylindrical record, was drilled on its exterior surface with holes about 1 mm diameter and 1 mm deep, arranged in rows circumferentially around the drum. The constant dot spacings, necessarily precise, in a given row, were easily obtained on a milling machine. The drum was then covered with dead black, followed

by a thin coat of clear lacquer. After the cylinder dried, a pasty mixture of precipitated chalk and water (the suggestion of Mr. Lewis Delsasso following upon difficulty with white lead) was rubbed over the surface and wiped off, leaving the holes filled with the chalk, and the surrounding blackened surface free from white. This gave a satisfactory stroboscopic screen of white dots on a black field. The screen was run on an Edison triple-spring recording phonograph. Uniformity in the motion was tested occasionally with a tuning fork, although in the present work great constancy of motion over extended periods was unnecessary.

The total number of rows of dots was limited to 15, distributed among three groups of five rows each. The stroboscopic effect itself gave the octave relation, so that a fourth group was unnecessary. The spacing between the rows in a group was about 3½ mm. The extreme rows of adjacent groups were spaced somewhat more than this, about 7 mm, for easy recognition of the individual groups. The arrangement of the rows may be indicated by the total number, N, of dots in each row, as follows:

30-31-*32*-33-34 38-39-*40*-41-42 46-47-*48*-49-50

It will be noted that the values of N for the three central rows are in the ratio, 4:5:6, that of the major triad.

THE STROBOSCOPIC RESPONSE

When a tone of given frequency is sounded, there accompanies it a characteristic appearance of the drum due to the stroboscopic response. This appearance can be discussed to better advantage after the situation has been defined algebraically.[5]

Let N represent as before the total number of dots around the drum in a given row; ν, the rotational rate of the drum in revolutions per second; n_d, the frequency of the actual dots in the row; n_t, the frequency of the sounded tone; and R, the number of stroboscopic dot images in the row, passing a fixed point per second. Then $n_d = N\nu = (R + n_t)/k$, where k is an integer. n_d is always to be regarded as positive if the direction in which the drum surface is moving is taken positive, which is desirable, so that $R \gtreqless 0$ according as $n_t \lesseqgtr kn_d$.

The dot frequencies, n_d, may be referred to the major diatonic scale, and they may then be spoken of as lying in a definite "key," dependent upon the rotational rate, ν, of the drum. The pitch to which the instrument is thus "keyed," is changed by changing ν, which can be done through small degrees by means of the speed regulator of the

[5] L. E. Dodd, Iowa Academy of Science Proceedings, *24*, p. 221; 1917.

phonograph. Thus the present arrangement of only fifteen rows of dots covers the range of the voice, at any rate the male voice.

From the equation above it is seen that the typical stroboscopic behavior of a series of rows of dots having from left to right increasing values of N, gives a central stationary row (assuming n_t and ν to be precisely adjusted so that $R=0$ for this row), and if it is supposed that the drum surface has upward motion, the dot images in the rows at the left of the apparently stationary, or "zero" row, will move downward at the rate R, this rate increasing as we go from row to row away from the zero row, while the dot images at the right will move upward, and at increasing rates as we go from row to row away from the zero row. In other words, there is a definite region, or "range," of stroboscopic response that is in a sense symmetrical. At its left margin, where the value of R is not too great for visibility of the moving dot images, a row of dots appears to move downward with a speed that for visibility of the response is a maximum. At its right margin another row of dots appears to move upward at maximum speed. Between these two extreme rows there appear rows of dot images having different speeds, ranging all the way from the minimum (or maximum negative) value, up through zero to the maximum (positive) value, because the value of R increases from row to row. Obviously, this type of symmetrical response is an aid both in assisting the subject to come to the correct initial pitch (where the tuning fork is not continually used), and in confirming judgment whenever necessary, as to the direction of any pitch change.

If $R=0$, the stationary response is present, as stated above, and $n_d=(n_d)_o=n_t/k$. Thus, k is seen to be the value of the ratio of the tone frequency to the dot frequency, and is unity for the zero row if the two frequencies are equal, and 2 if the tone frequency lies an octave above the dot frequency. The constant, k, then retains, over the whole of this "range" of stroboscopic response, the particular value it has for the zero row. Writing the subscript zero when referring to the central stationary, or zero, row, $k=n_t/(N_o\nu)$.

Letting d represent the distance of separation of the actual dots in a given row, $d=2\pi r/N$, and if d' is the separation distance of the dot images for that row, $d'=d/k$, so that $d'=2\pi r/(kN)$. For the zero row, $d'=2\pi r\nu/n_t$.

Now regarding one of the groups of five rows each on the present drum screen, and supposing that the central row of the group is stationary by stroboscopic response, the increment Δn_d from row to row, left

to right, is positive, constant, and equal to ν. The increment in R is positive, constant, and equal to $k\nu$, or, $\Delta R = k \cdot \Delta n_d$. Further, the increment in d' from a given row to the one next it on the right, is $\Delta d' = (-2\pi r/k) \cdot [1/N(N+1)]$, where N is the total number of dots in the first of the two rows. Thus, d' decreases from left to right as N increases, and its increment is not constant, but depends upon N.

The maximum numerical value of R for visibility of the series of stroboscopic dot images in a particular row depends upon a number of different factors, including (1) the separation distance, d, of the actual dots (as distinguished from the stroboscopic dot images) in the row; (2) the rate, R, of their motion; (3) the separation distance, d', of the dot images; (4) the size of the actual dots, particularly relative to their separation distance, d; (5) the visibility of the actual dots themselves (a question of color and brightness values of the dots as compared with their background); (6) the manner of production of the stroboscopic response, including details of a particular method; (7) the brightness of the illumination; and (8) the individual observer (personal equation). In the present work the separation distances, d', of the stroboscopic dot images were sufficiently large so that they were seen clearly distinct and separate with the stationary response, thus permitting a clear indication of the direction of the pitch change.

TABLE 1. *Numerical values of quantities involved in the stroboscopic response.*

Group	N_o	$\nu = \Delta n_d$	$(n_d)_o$	k	ΔR	$(d')_o$(mm)	$-\Delta d'_o$(mm)
				1. $n_1 = 128$			
1	32	2.000	64	2	4.000	2.76	0.08390
				4	8.000	1.38	.04195
2	40	3.200	128	1	3.200	4.42	.10791
				2	6.400	2.21	.05395
3	48	2.666	128	1	2.666	3.68	.07531
				2	5.333	1.84	.03765
				2. $n_1 = 214$			
1	32	3.344	107	2	6.687	2.76	0.08390
				4	13.375	1.38	.04195
2	40	2.675	107	2	5.350	2.21	.05395
				4	10.700	1.10	.02697
3	48	2.229	107	2	4.458	1.84	.03765
				4	8.917	0.92	.01882

In Table 1 are values of quantities involved in the stroboscopic response, covering all practical cases in the present study, although not every tabulated case was used in the tests. The zero subscripts again refer to the stationary state. The three groups of rows of dots are treated individually, with specific reference to the central row of each group. The increments apply, as already mentioned, to the successive rows in a given group, but mathematically could be extended to other actual or possible groups.

The voice pitches (values of n_t) employed ranged from C_2128 to A_2214. As the maximum speed of the drum was somewhat less than 4 r.p.s., it was necessary at times to substitute the 2:1 ratio ($k=2$) of the tone to the dot frequency, for the simpler 1:1 ratio ($k=1$). This was done by adjusting the motor speed. Since the dot images are closer together the larger the value of k, this quantity is chosen not larger than necessary to bring ν below ν_m, where the latter symbol represents the maximum attainable drum speed. If we designate the drum frequency for a particular stroboscopic range (particular value of k) by ν_k, then for the stationary row, $\nu_k=n_t/(kN_o)$. Then ν_k is chosen so that $\nu_k<\nu_m<\nu_{k-1}$.

If the screen is observed at the center of the path of the oscillating beam, as was usual, rather than at, or near, its position of extreme displacement, the illumination frequency is doubled, and the values of k are likewise doubled, which accounts for $k=4$.

Incidentally, an approximate idea of the precision with which ν can be adjusted, is obtained from the relation $\nu=(R+n_t)/(kN)$. Regarding n_t and N as constant, we have $\Delta\nu=\Delta R/(kN)$. Experimentally, by the use of a tuning fork to insure constancy of n_t, the speed of the motor can be adjusted so that $\Delta R \not> 0.1$, say, which is an estimate, but a conservative one. Taking the extreme values of N and all values of k having to do with the present investigation, we get the corresponding values of $\Delta\nu$, Table 2. The values of d' for the six cases are included in the table.

TABLE 2. *Precision of speed adjustment of motor.*

k	N	$\Delta\nu$	d'(mm)
1	30	0.00333	5.900
	50	.00200	3.544
2	30	.00166	2.941
	50	.00100	1.772
4	30	.00083	1.470
	50	.00050	0.886

The first of the two extreme rows of dots dealt with in this table, belongs to group 1, and the second to group 3. The maximum value of ν actually used with group 1 in the tests, could not have been below 2.00, nor with group 3 below 2.23, as reference to Table 1 will show. For group 1 the corresponding value of $\Delta\nu$ as given in Table 2 is 0.00166, and that for group 3 is 0.00200. The corresponding values for the precision of adjustment of the motor speed to agree with a tuning fork frequency, stated in terms of percentage variation per second in the rotational frequency of the drum, when computed on this basis, are then, 0.083 and 0.089 respectively, values appreciably less than 0.1 per cent. The speed constancy of the drum is a question distinct from that of precision of speed adjustment, but as a matter of fact it is involved with the latter, so that the foregoing percentages include both variations.

In the discussion above it was assumed that through the use of a tuning fork the value of n_t could be regarded as perfectly constant. Inquiry will next be made into the precision with which change in n_t, the frequency of the sounded tone, can be detected. To do this we regard ν as constant. From the equation given above, $n_t = kN\nu - R$. Fixing attention on a given row, N, we have $\Delta n_t = -(R_2 - R_1) = -\Delta R$, so that the pitch change and the change in R are oppositely directed. If the row is initially stationary by stroboscopic response, $R_1 = O$, and $\Delta R = R_2$, and if the pitch of the tone now rises, the row will move downward, while if the pitch drops, the row of dot images will move upward. Not only must the effect upon R due to a pitch change caused by the abruptly applied pressure on the throat be considered, but also any change due to the subject's not being able to maintain the pitch of his tone constant in the absence of such pressure. However, the subject had little difficulty maintaining the tone pitch constant to one or two vibrations per second over the necessary short period of a few seconds. At any rate, the abrupt effect due to the pressure was clearly discernible, even when the observed row slowly moved initially. Putting $\Delta R = 1$ as the assumed least detectable value of the abrupt effect, whether the observed row is stationary or slowly moving, then the least detectable abrupt change in the pitch due to the pressure would be unity also. out of the pitch changes definitely recorded as such in the present data, practically all of them were much greater than this. It is a conservative judgment that the value of ΔR, likewise the value of Δn_t, due to the

pressure, was not less than 3 in any case recorded as that of a pitch change, and ranged from this up to 10 and above. At times it amounted to a whole tone or more.

In the foregoing remarks it was supposed that the value of d' did not seriously affect the observer's judgment of the fact of pitch change. Judgment as to the direction of the pitch change is an incidental matter of very little uncertainty.

The values of n_d for the five rows in a group change from row to row by increments, Δn_d, considerably greater than unity. This means that R also changes from row to row by correspondingly large increments. In fact, ΔR might be so great that the stroboscopic response is not recognizable even in the two rows immediately adjoining the zero row on either side. For the present work this could have been of no great disadvantage, since a knowledge of only the direction of the pitch change, and not its amount, was desired. For the sake of discussion let it be assumed that the dot images are no longer discernible as such for values of R above 10, say. Then there are only two cases of the twelve given in Table 1, where the response is no longer to be recognized in the two rows on either side the zero row. These two cases were easily avoidable by employing group 3. But even in these two cases, if the tone pitch changes somewhat, either the first row to the right of the central row, or that to the left, will begin to respond visibly, depending upon whether the pitch is rising or falling. For instance, if the pitch should gradually rise from $(n_t)_o$, the dot images of the central row will start to move downward, increasing their stroboscopic motion with farther pitch change, while the upward motion of the dot images in the first row to the right, initially too rapid, as assumed, for the response to be seen, will decrease and approach zero, at which time the stationary state for this row will have been reached, and so on if pitch change in the same direction continues. Thus, the fact that either one of the rows at the right of the center is seen to respond, means pitch change upward. In this way, even for the two extreme cases, the rows on either side the zero row can still assist in determining the direction of the pitch change. In the other ten cases one or both rows on each side of the central, zero row, give the symmetrical type of response, and not only the behaviour of the central row, but also the general response over several rows at the same time, can be observed readily.

Obviously, if the observer determines the direction of the pitch change from a single row of dots, any one of the rows in any group might be used, provided it appears stationary, or approximately so, when the subject sounds the tone initially.

As to the sensitiveness and the visibility of the stroboscopic response by this arrangement compared with the manometric flame, no extensive quantitative study has been made. But in sensitiveness of response, at any rate, it was found far superior to the manometric flame at hand. However, a flame has the advantage of giving the response over a larger area on the stroboscopic screen, but not necessarily over a greater distance measured perpendicular to the parallel rows of dots as usually employed on such a screen. Further, the frequency of the periodic flashes of the flame equals the frequency of the sounded tone, while a mirror actuated by a vibrating diaphragm gives twice that frequency of the flashes in a limited region on the stroboscopic screen containing the equilibrium position of the beam. The disadvantage in the latter case is that the stroboscopic images appear twice as close together, assuming that the tone and the dot frequencies are equal, and closer together still for octave-above relations of the tone to the dot frequency, reducing somewhat the visibility of the response at higher pitches. This limitation can be diminished either by speeding up the screen, or by enlarging it so that the dot spacing is greater, or it can be removed entirely by observing the limited region on the screen, where the vibrating beam reaches its maximum displacement on either side of the central, or equilibrium, position. But regarding this alternative, the extreme displacement position varies with the loudness of the tone and also with the degree of the diaphragm response, and further is dependent upon resonance effects in the horn. In the present work observations were therefore made usually at the central position of the beam on the screen.

PROCEDURE, RESULTS, AND CONCLUSION

Briefly, the procedure in the present application is for the subject to sound and sustain a tone of moderate loudness on a prescribed pitch, so that by stroboscopic response one of the parallel rows of dots on the drum, preferably the central row of a group, appears stationary when the drum is rotating at a fixed speed known by means of a tuning fork. The initial pitch of the sounded tone is tested stroboscopically by the subject himself. The somewhat abrupt pressure was applied by means of the finger tip, a desirable method because of the certainty of the particular locality on the throat being subjected to the pressure. The direction of any resulting pitch change is then noted as above described.

To simplify the tests, the localities chosen for applying the pressure were restricted to the vertical median anterior-posterior plane, and

thus lay along a line following from the angle beneath and back of the chin down to the farthest accessible point on the trachea, in the U-shaped depression above the breastbone. Ten localities were selected for the first tests, made on three different individuals, using the eight vowel sounds of Miller's pyramid.[6] These ten localities were: hyoid, hyo-thyroid, thyroid, or Adam's apple (tip, center, and base), thyro-cricoid, cricoid, and trachea (three different places). As soon as a characteristic type of response as to pitch change became apparent, the number of localities for application of pressure was reduced to three: viz., thyroid (tip), cricoid, and trachea (center of the accessible region).

Since a detailed presentation of data obtained would fall outside the scope of this journal, the results will merely be summarized here, and the data in detail included in an article to be submitted to a publication of special nature more appropriate to a discussion of the results. Three men subjects were studied, using the eight vowel sounds mentioned, each vowel on the same two pitches, making six cases in all, as follows: tenor, E_2160, A_2214; baritone, C_2128, F_2171; and bass, C_2128, F_2171. The results showed consistently for all vowel sounds a drop in pitch by pressure on either the thyroid cartilage or the trachea between the cricoid and the breastbone, while pressure on the cricoid cartilage itself showed in five out of the six cases a consistent rise in pitch, and in the sixth case, either a rise or a tendency to maintain the pitch in spite of the disturbing pressure.

Although the amount of the pressure used in the tests upon which the generalization above is based, was kept fairly uniform, no quantitative data were obtained relating magnitude of pressure to magnitude of pitch change. The generalization is therefore rather qualitative than otherwise, but qualitative in a precise sense. Quantitative work of the kind will be worth while.

However, a simple dynamometer for applying pressure to the throat and registering its amount, was constructed incidentally toward the close of the present tests. It consisted of a plunger that moved backward against a helical spring held inside a brass tube, when the outer end of the plunger was pressed against the throat, the inner end in turn pushing out of the far end of the tube another plunger calibrated after the manner of an air gauge for automobile tires. This dynamometer was used on the bass, and the pressure necessary for unmistakable pitch change (say 2 v.p.s. and above) was found to be of the order of

[6] The Science of Musical Sounds. p. 230.

100 grams. For the other subjects it would probably have been found considerably less than this.

Any conceivable effect upon a sustained vocal tone due to any cause whatever, must be a change in any one, or a simultaneous change in any two, or all three, of the characteristics of a musical tone. Although the ear is quite sensitive to detect change in a sustained tone, it can hardly be relied upon to differentiate slight changes in these characteristics, especially where changes in all three occur at the same time. The present method takes note only of pitch. It is a visual test of an auditory effect, so that the test is objective in character, a decision being reached on the basis of what is seen, not what is heard.

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JULY 1925.

Foundations of the Universe. By M. Luckiesh. New York: D. van Nostrand Co. 1925.

This book consists of an introductory chapter; of a series of chapters successively devoted chiefly to dynamics, kinetic theory, light, the ether, light again, relativity special and general, the periodic table, the electron, radioactivity, atomic structure, light-quanta, atomic structure again; "The Fateful Unknown"; a chronological list of eminent thinkers and experimenters from Thales to Millikan, with a few words about achievements of each; and a tabulation of definitions and numerical constants.

The book belongs to a series described as "devoted to the popular treatment of scientific subjects" and hence perhaps cannot be properly judged by anyone whose specialty is physics. If my judgment is at all nearly correct, it must prove difficult reading for a "layman," partly because of the quantity of information—for it is very detailed as well as wide-ranging—and partly because technical words for unfamiliar things and ideas are introduced early and suddenly (e.g. "electromagnetic energy" on page 1, "kinetic energy" at the beginning of the chapter on kinetic theory without any prior definition, etc.). The audience for Dr. Luckiesh's book must therefore be drawn from among people who studied physics not very long ago and have remembered their studies well. There are inaccuracies which should be corrected: for instance, the statements that the accuracy of the Michelson-Morley experiment was one part in four billion (actually a shift 1/6 as great as that which would result from a stationary ether could not have been perceived); that water molecules in electrolysis divide into positive hydrogen and negative oxygen ions (instead of H^+ and OH^-); that diamond, graphite and amorphous carbon differ in the arrangement of atoms in the molecule; the implication that light-pressure is theoret-

ically a consequence of the electromagnetic theory in particular instead of undulatory theories in general, etc.

Dr. Luckiesh begins admirably by trying to impress on his readers the two important principles that the new and strange is not necessarily more wonderful than the old and familiar, and that science is not necessarily antagonistic to appreciation of beauty. His emotional attitude toward science will appeal variously to various readers, but is no doubt better adapted to make converts to scientific studies than is the colorless style of many scientists. It should be frankly stated that while many passages are written in a clear and fluent manner, many others are very carelessly composed.

KARL K. DARROW

Photoelectric Effect of Thin Films of Sr.—Thin films of Sr were formed by sublimation of the metal onto a glass plate in vacuo (no indication of their thickness is given); and they were found to display a rather pronounced selective effect, in both senses of the word. That is to say: (a) the photocurrent depended on the orientation of the electric vector of the incident plane-polarized light, being greater, the greater the component of the said vector normal to the surface of the film; and (b) the curve representing the photocurrent when the electric vector was oblique to the surface, reduced to unit incident intensity and plotted versus wave length, showed a high rounded maximum near 340 $m\mu$. The currents produced were very capricious, and in general diminished steadily and rapidly from the moment that the deposition of the film was halted; so also did the ratio of the currents produced when the electric vector was oriented successively parallel and normal to the plane of incidence. This ratio, which serves as a measure of the selective effect in the first sense, attained values as high as 7.7 when the measurement was made very quickly (2 minutes) after ceasing to sublime metal over onto the film; in 20 minutes it sank (in one case) from 7.7 to 3.7. Selective effect of the same general magnitude was observed with films sublimed in hydrogen or in water-vapor. [R. Döpel, Göttingen; ZS. f. Phys., *33*, pp. 237-245; 1925.]

KARL K. DARROW

Speculations on the Relative Intensities of Lines constituting Bands.—Starting from Burger's and Dorgelo's rule that in a multiplet the sum of the intensities of all lines originating from one initial state is proportional to the *a priori* probability of that state, (and likewise for all lines terminating in one final state), Dieke applies them to band-spectra. He assumes that the *a priori* probabilities of the various stationary states, in which the angular momentum of the molecules is equal to various multiples $jh/2\pi$ of $h/2\pi$, are proportional to $2j-1$; and deduces quite definite predictions as to the relative intensities of the lines, for which, however, experimental material is not yet available for testing. [G. H. Dieke; ZS. f. Phys., *33*, pp. 161-168; 1925.]

KARL K. DARROW

A DOUBLE PERCENTAGE BRIDGE

By A. C. Longden

ABSTRACT

Description. This instrument is essentially a double bridge supplied with two slide wires which are used in equalizing the ratio coils. The resistance of the ratio coils bears such a relation to that of the slide wires that the ratio coils may be used as the auxiliary coils in a percentage bridge.

Advantages. In addition to simplifying the process of equalizing the ratio coils, this device offers a simple and exact method of balancing the bridge without shunting either of the standards or resorting to the rather unsatisfactory method of interpreting galvanometer deflections.

Some suggestions are given concerning details of construction.

In view of the difficulties involved in adjusting the ratio coils of a Kelvin double bridge, and also in balancing such a bridge, the writer of this paper has designed a bridge which he has chosen to call a double percentage bridge. This bridge combines the precision of the Kelvin double bridge in low resistance measurements with the simplicity and convenience of the percentage bridge.[1]

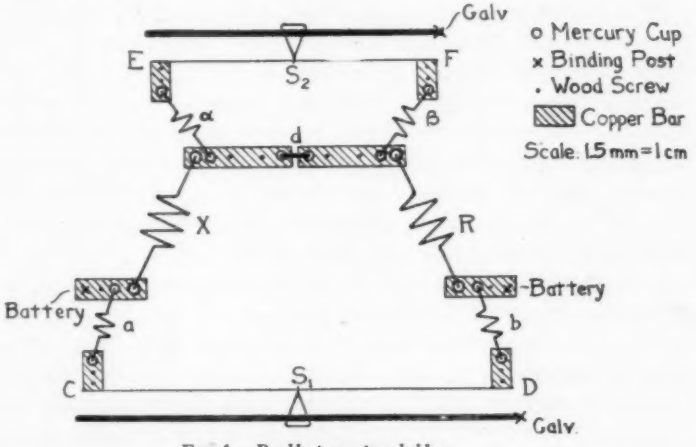


FIG. 1.—*Double percentage bridge.*

The accompanying figure illustrates the principles involved as well as some of the details of construction.

[1] Phys. Rev., *24*, p. 349; 1907.

In this figure, a and b are the main ratio coils, α and β the secondary ratio coils, X and R the standard resistances under comparison and d, the connecting link. CD and EF are bridge wires provided with sliding contacts S_1 and S_2. The battery and galvanometer are connected as shown.

An inspection of the figure will show that the six principal conductors and the connecting link, d, constitute the essentials of a Kelvin double bridge; and further consideration will reveal the fact that the addition of the two slide wires affords a unique and simple method of equalizing both sets of ratio coils and also an exact method of balancing the bridge without the necessity of shunting either of the standards or of interpreting galvanometer deflections in comparing them.

To equalize a and b, remove X and R and replace them by heavy copper links of negligible resistance as compared with that of the ratio coils. This will not be difficult, as all of the ratio coils have relatively high resistance. Next, remove the connecting link, d, and the network of conductors is converted into a four gap Wheatstone slide wire bridge in which CD is the bridge wire. The two parts of the wire EF into which it is divided by the slider S_2 may be considered as being temporarily added to α and β.

With this arrangement, a and b may be compared with each other by the Carey-Foster method, using CD as the bridge wire and S_1 as the slider. The two balancing points may not be very near the middle of the bridge wire, because of inequality between α and β; but the resistance of the portion of the wire between the two balancing points is the true difference between a and b. If the slide wire has been previously calibrated and found uniform, the slider may now be set at half the distance between the two balancing points on the proper side of the geometrical center of the wire. This position of the slider will be the true electrical center. That is, a and b are equalized by adding suitable portions of the wire CD to each of them.

This true electrical center is now the zero point from which subsequent readings on the wire are to be made. For the sake of convenience in reading, a scale, reading in both directions from a central zero, may be reset with its central zero at the new position of the slider.

As the network of conductors is symmetrical, the slider S_1 may be left in its new position, and with EF as the bridge wire and S_2 as the slider, α and β may be equalized by using the method just described for a and b.

The connecting link, d, and the resistors X and R may now be replaced and X may be compared with R as with a Kelvin double bridge, except that the bridge wires and sliders may be used in balancing the bridge. The balance is obtained by simultaneously adjusting the two sliders. If the linear resistance of the bridge wires bears the proper relation to the resistance of the ratio coils, the percentage method already cited may be used.

As a double percentage bridge, a and b are the auxiliary coils for the wire CD and α and β are the auxiliary coils for the wire EF. In order that the proper percentage relation shall exist, each auxiliary coil must have a resistance equal to 19.5 times the resistance of a meter of the bridge wire. The wire need not be a meter long, but the sum of the three resistances, a, b and the wire CD must be $40{,}000\rho$, where ρ is the resistance of a millimeter of the wire CD. Likewise, the sum of the three resistances α, β and the wire EF, must be $40{,}000\rho$, where ρ is the resistance of a millimeter of the wire EF.

If the resistance of the ratio coils a and b is double that of the ratio coils α and β, as is often customary in double bridge construction, then the sum of the three resistances, a, b and the wire CD should be $80{,}000\rho$, and the scale accompanying the wire CD should be provided with double spacing. This is the reason for making the wire CD longer than EF, as shown in the figure.

The substitution method is used in connection with this bridge, just as in the ordinary percentage bridge, the gap between a and α being used as the substitution gap.

To make a measurement, R_1, the standard with which X is to be compared, is placed in the substitution gap and the bridge is balanced. X is then substituted for R_1 and the bridge is again balanced. Then X equals R_1 plus or minus the difference in reading between the two positions of either slider, interpreted in hundredths of one per cent. If the difference is large, a correction should be made for distance from the middle of the wire, as described in the original Physical Review article on The Percentage Bridge. However, in the comparison of fairly exact standards, differences as large as .1 per cent are never met with, and in such cases no correction is necessary.

It must be remembered that S_2 is read in millimeters and S_1 in double millimeters, or fifth of centimeters, if a and b have double the resistance of α and β.

The use of the substitution method will eliminate the spurious resistances between the standards and the ratio coils, and since all the

other spurious resistances are eliminated in equalizing the ratio coils, the instrument is one of very great precision.

KNOX COLLEGE,
GALESBURG, ILL.
JULY 10, 1925.

Table of Wave Numbers. **H. Kayser, Professor of Physics, University of Bonn, 106 pp; S. Hirzel, Leipzig, 1925; price unbound M10.80, bound M13.**

This exceedingly useful and important table enables one to convert directly from wave lengths measured in air to wave numbers, i.e., reciprocal wave lengths in cm corrected to vacuum. The wave lengths are tabulated in 0.1A from 2000 to 7000A and by Angstroms from 7000 to 10000 with the corresponding wave numbers arranged similar to a logarithm table. Interpolation tables accompany the main body of the table so that one may readily obtain the wave number to 8 figures of a wave length expressed to 0.001A. For example, by a simple interpolation the iron line λ3850.821 in air is found to have a wave number 25961.158. The reduction to vacuum is based on the refractive index measurements of Meggers and Peters at 15°C and 760 mm pressure.

The immense amount of work required to prepare these tables is appreciated when one considers that it was first necessary to obtain the reduction factors for each whole Angstrom, interpolate these for tenth Angstroms and then reduced the 64000 numbers to vacuum wave lengths. These were finally converted to reciprocals of eight figures by use of a Mercedes-Euclid electrically driven computing machine. Of course, many checks were made to avoid errors in the original manuscript and in its reduction to print.

The following paragraph from the preface is a tribute to its author who is at present engaged in the revision of his monumental "Handbuch":

"Since a very large and constantly increasing number of research workers is concerned with wave number computations it is apparent that a vast amount of time is wasted. For the interest of science it therefore appeared desirable that I use *the spare time* in my advancing age to prepare a table in which wave numbers could be read directly from wave lengths." The spectroscopic world is indebted to Prof. Kayser for his "spare time work" and to Hirzel for making it accessible at such a reasonable price.

PAUL D. FOOTE

FURTHER EXPERIMENTS ON THE THEORY OF THE VANE RADIOMETER

By H. E. Marsh

INTRODUCTION

In a previous communication[1] it was shown that the prediction of the Einstein[2] theory of the vane radiometer, that the radiometric effect was proportional to the edge moment of the vane rather than to the area moment as had been previously supposed, was probably correct. In these earlier experiments certain discrepancies with the theory were noted, viz., that (1) the effect was not quantitatively correct, and (2) that the sensitivity of the radiometer varied with the pressure contrary to the Einstein theory. It was suspected that the first deviation from the theory might be due to the fact that the temperature difference between the illuminated and the unilluminated faces of the vane for the edges of the vane with the large edge moment might be less than for the vane with the small edge moment, owing to the greater cooling effects where the edge moments were great. The second deviation could not be explained at the time. It was with the view to eliminate these difficulties, or explain them, that the later experiments were undertaken. In anticipation, it may be stated that despite the crudeness of the measurements the results indicate that both deviations are due to temperature effects which, when eliminated, would lead to a verification of the theory. The results, furthermore, lead to an evaluation of the radiometric forces acting with results which are in good agreement, in order of magnitude, with the predictions of the Einstein theory.

This paper contains the results of the following investigations: (1) The initial experiments were repeated with a more nearly homogeneous light source and more nearly ideal conditions, together with experiments on a new design of vane which had, on the two sides of the vertical axis, equal area moments but with edge moments in the ratio of two for the grid side to one for the solid side. (2) In order to investigate the pressure variations, a new radiometer chamber was used in which, by the elimination of grease joints, much lower pressures could be employed. For this chamber a new vane was required having half the

[1] Marsh, Condon, and Loeb, J.O.S.A. & R.S.I., *11*, p. 257; 1925.
[2] A. Einstein, Zs. f. Phys. *27*, p. 1; 1924.

dimensions of the old ones and a ratio of edge moments of 1.6 to 1 while the area moments on the two sides of the vertical axis were the same. (3) The question concerning the temperature differences involved was settled by actual temperature measurements on the illuminated and unilluminated faces of the vanes and on the different parts of the illuminated face of the vane as a function of pressure, using very small thermocouples. (4) The theory was tested by actually measuring the forces exerted on the vanes, when illuminated, through a measurement of the torsional constant of the fibres.

APPARATUS

Description of Chamber No. 1.—The vanes were suspended by means of a quartz fibre from an adjustable brass support, the entire device being enclosed in a bell jar, as shown in Fig. 1. In the later experiments

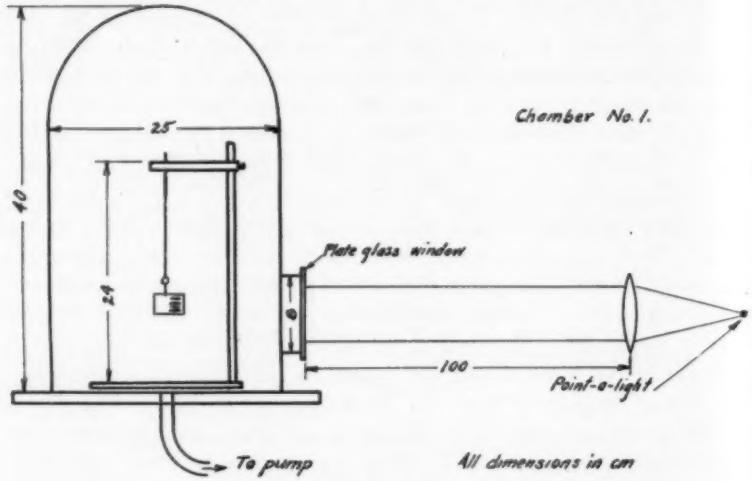


FIG. 1.—*Arrangement of vacuum chamber.*

a circular hole was cut in the side of the bell jar and a plate glass window mounted on the same with sealing wax, thus allowing the beam to come to the vane without much distortion.

Reduced pressures were obtained by means of a mercury vapor pump backed by an ordinary motor driven vacuum pump. Pressures were measured by means of an accurately calibrated McLeod gauge reading to thousandths of a millimeter of mercury.

Description of Chamber No. 2.—Since it was necessary to use a grease joint in the chamber described above, it was impossible to get pressures lower than a few hundredths of a millimeter. In order to secure lower pressures a second chamber was designed as shown in Fig. 2. This chamber consists of a brass cylinder ten centimeters in diameter and ten centimeters long. The ends were ground on a face plate to receive plate glass windows which were secured by means of sealing wax. Brass tubes were soldered into the upper and lower sides of the chamber to allow for the suspension of the vane and the evacuation of the chamber. This chamber made it possible to secure pressures as low as one thousandth of a millimeter.

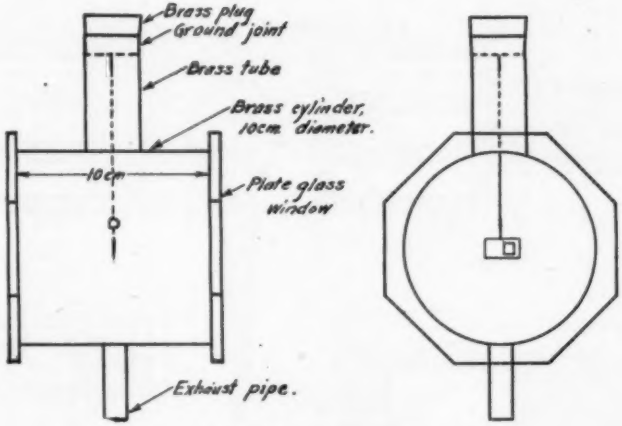


FIG. 2.—*High vacuum chamber.*

Source of Radiation.—The source of radiation was a five hundred candle power "point-o-light" and was placed at the focus of a 10 centimeter lens thus insuring a homogeneous beam of parallel rays. The homogeneity of the beam was insured by means of measurements with a thermopile. A diaphragm was placed in the path of the beam so that either end, the entire surface, or none of the vane could be illuminated as desired. The experiments were carried on in a darkened room and the bell jar was covered so far as possible in order to reduce the stray light to a minimum.

Measurement of Deflections.—The deflections were measured by means of a small beam of light reflected to a transparent scale from a light mirror attached rigidly to the vane (see Figs. 1-4).

Description of Vanes.—Vanes of four different designs were used and are shown in Figs. 3, 4, 5, 6. Two vanes of each design were made of mica, one 0.45 mm thick and the other 0.05 mm thick. In each of these vanes the area moments about the vertical axis were the same. In Vane No. 1 (Fig. 3) the edge moments were in the ratio of about four for the grid side to one for the solid side; in Vane No. 2 (Fig. 4) about three for the grid side to one for the solid side; in Vane No. 3 (Figs. 5-6) about two for the grid side to one for the solid side. Beside these four vanes a control vane of mica, 0.45 mm thick and having perfect symmetry about the vertical axis, was used (see Fig. 7). A vane of design No. 1 was also made of aluminum foil about 0.05 mm thick.

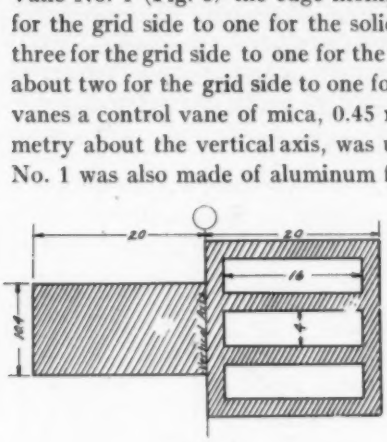


FIG. 3

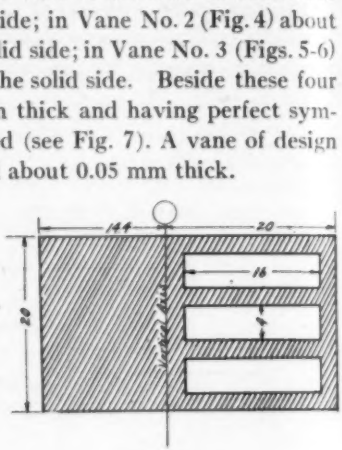


FIG. 4

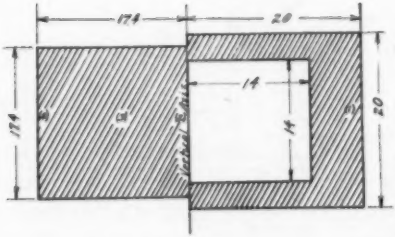


FIG. 6

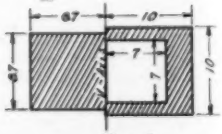
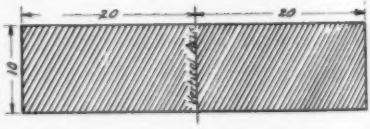


FIG. 5



All dimensions in mm

FIG. 7

Types of Vanes

Temperature Measurements.—For measurements of temperature differences a pair of thermocouples was devised as shown in Fig. 8. This pair consisted of a piece of constantan wire about ten centimeters long and 0.0386 mm in diameter, having its two ends joined to two pieces of platinum wire 0.05 mm in diameter, thus having a very low heat capacity and a fairly high sensitivity. The two junctures were rigidly attached to the two ends of a glass "hair pin" which permitted

them to be crowded over the vane so as to be in close contact with the illuminated and unilluminated faces for the measurement of ΔT. When measurements of the temperature difference between different points of the *illuminated* face were made, the "hair pin" was spread to bring the junctures to the desired positions and then brought into close contact with the blackened face of the vane.

Calibration of Thermocouple.—The couple was calibrated by placing one juncture in melting ice and the other in water, the temperature of which could be gradually changed. A Leeds and Northrup galvanometer of high sensitivity was used and deflections read at a scale

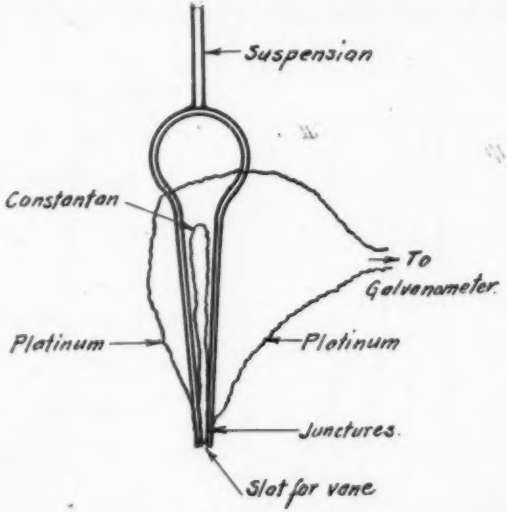


FIG. 8.—*Thermocouple.*

distance of 100 centimeters. From a series of readings, using mercury in glass thermometers, a calibration curve was plotted and the value of 0.08°C per millimeter of scale deflection was obtained.

Procedure of Pressure Measurements.—In the investigation for the relation between sensitivity and pressure, readings of deflections at various pressures were taken with the grid alone illuminated, with the solid portion alone illuminated, and with both the grid and solid portion illuminated. Readings for the zero point were taken at each pressure with entire face of vane darkened. All readings, both for sensitivity and temperature, were made in the order of decreasing pressures.

Determination of Torsional Constant.—To determine the torsional constant, a disk, having a diameter of 12.6 millimeters and weighing 0.1857 grams, was suspended by a piece of the quartz fibre, having a length equal to the average length employed in suspending the vanes, and the period of oscillation measured. From this value, together with the known length of the fibre and computed moment of inertia of the disk, the torsional constant was found.

RESULTS

1. *Radiometric effects as a function of the pressure.*—The results obtained under the new conditions are summarized in the curves shown in Figs. 9, 10, 11, and 12 for vanes of designs numbered 1, 2, 3, and 4,

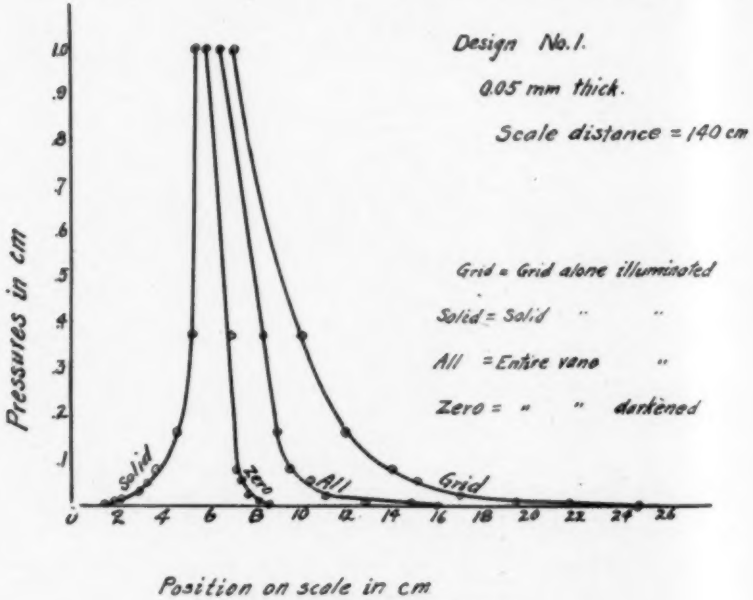


FIG. 9.—*Deflection-pressure curves.*

each 0.05 mm thick. It will be seen at once that the end of the vane having the greater edge moment (grid end) is deflected more than the other (solid end). The difference of the deflections of the two ends, in general, equals the resultant deflection with the entire vane illuminated. The relative deflection towards the grid and solid ends in the case of designs 1 and 2 is, however, (as was previously found) not in proportion

to the ratio of the edge moments but less than this. This condition can be ascribed to the fact that the vanes with the greater edge moment

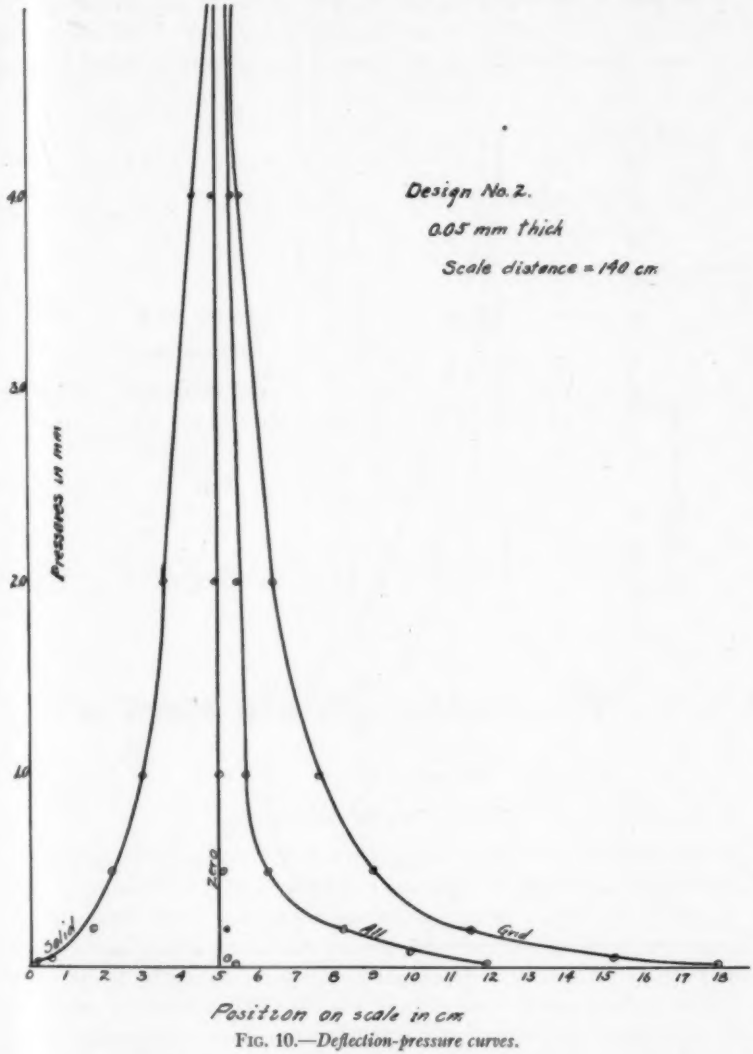


FIG. 10.—*Deflection-pressure curves.*

ratio have a distinctly lower temperature difference on their grid ends than those with the smaller edge moment ratio. In the case of the vane

of design No. 3 the ratio of the deflections is nearly that predicted from the ratio of the edge moments, viz., two to one. In this case the cooling of the grid is not so great as in the case where the grid had a greater edge moment. It is further seen that the sensitivity of the radiometer increases about four fold as the pressure decreases from about 0.4 mm

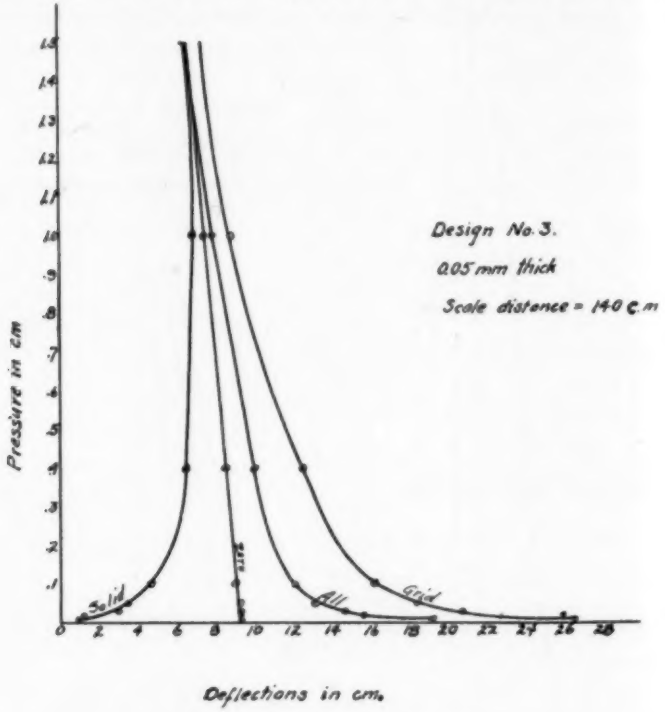


FIG. 11.—*Deflection-pressure curves.*

to 0.04 mm. In the range investigated there is no sign of a maximum effect such as would be expected from the results of Westphal. The change of sensitivity with pressure was not all due to the change in the mean free path and the nature of the radiometric action as originally suspected. It is largely explained by an increase in the temperature difference between the two sides of the vane as the pressure decreases. Experimental measurement of the temperature differences, which determine the magnitude of the deflections on the Einstein theory, showed that the temperature difference changed in a ratio of approxi-

mately one to three as the pressure was decreased through the range used in these experiments while the sensitivity only changed in the ratio

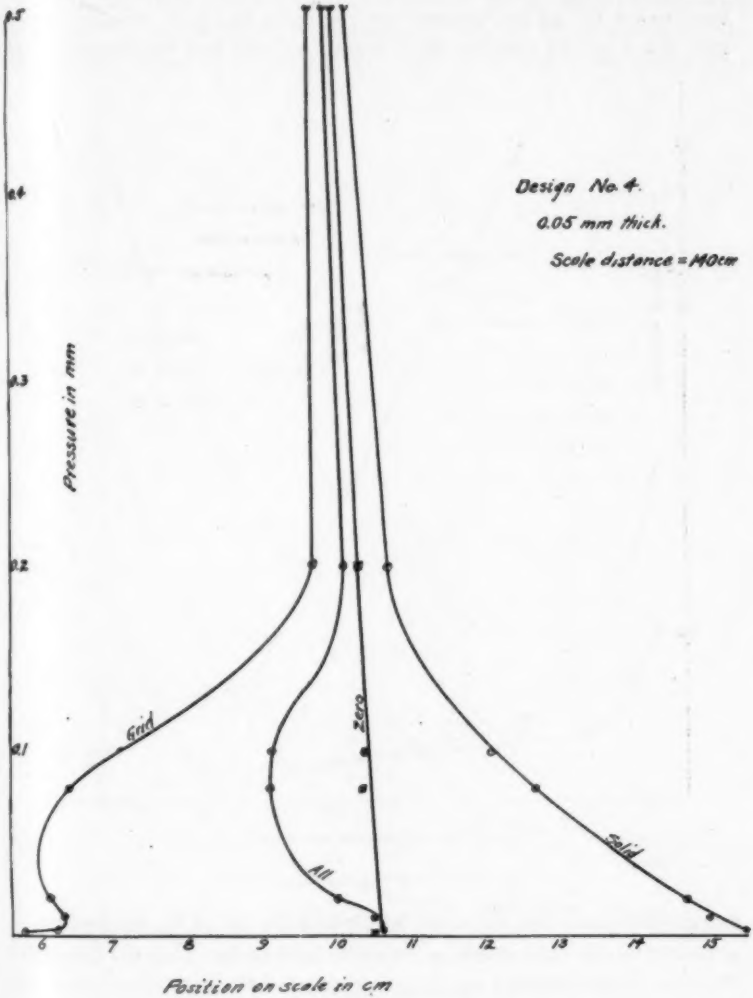


FIG. 12.—*Deflection-pressure curves.*

of one to four. Thus the increase in sensitivity observed is almost entirely accounted for by the rough temperature measurements made.

In the case of the experiments with the small chamber and very much lower pressures the deflections reach a maximum and then decrease, finally reaching a constant value. This indicates that, as the pressure went below the values used before, the mean free path became so large that the Einstein type of effect ceased to hold and *the direct reactions*

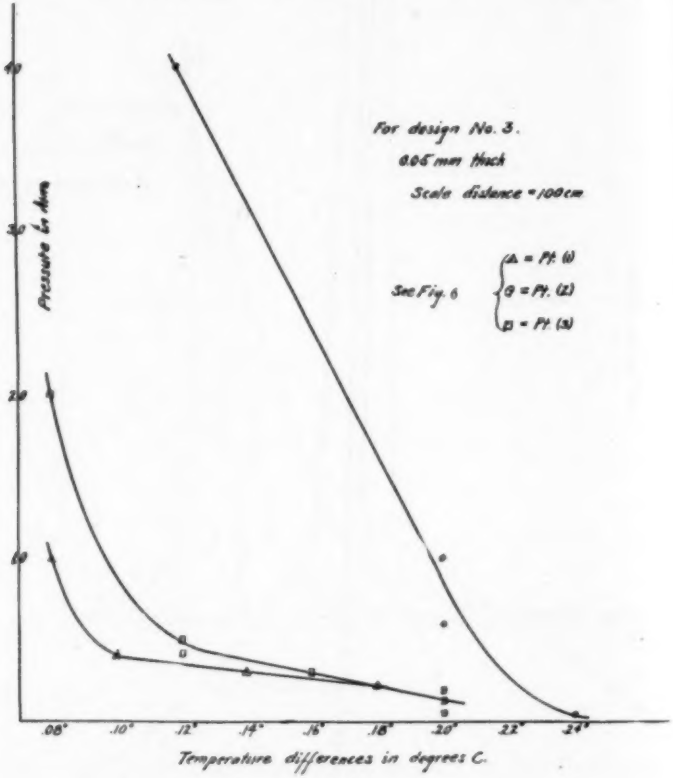


FIG. 13.—*Temperature-pressure curves.*

between the vanes and the walls began to enter in. It is interesting to note that as this regime sets in the effect of the edge moments disappear, the area moments being the effective cause of the deflection. The drift of the zero in experiments may be due to the action of the light on the mirror or to some stray light effect which becomes more important as the pressure decreases and temperature differences increase.

TEMPERATURE MEASUREMENTS

a. *On illuminated and unilluminated faces of vanes.*—The measurement of the temperature differences between the blackened illuminated side and the unblackened unilluminated side at three points on the vanes, designated as points (1), (2), and (3) in Fig. 6, are given for a

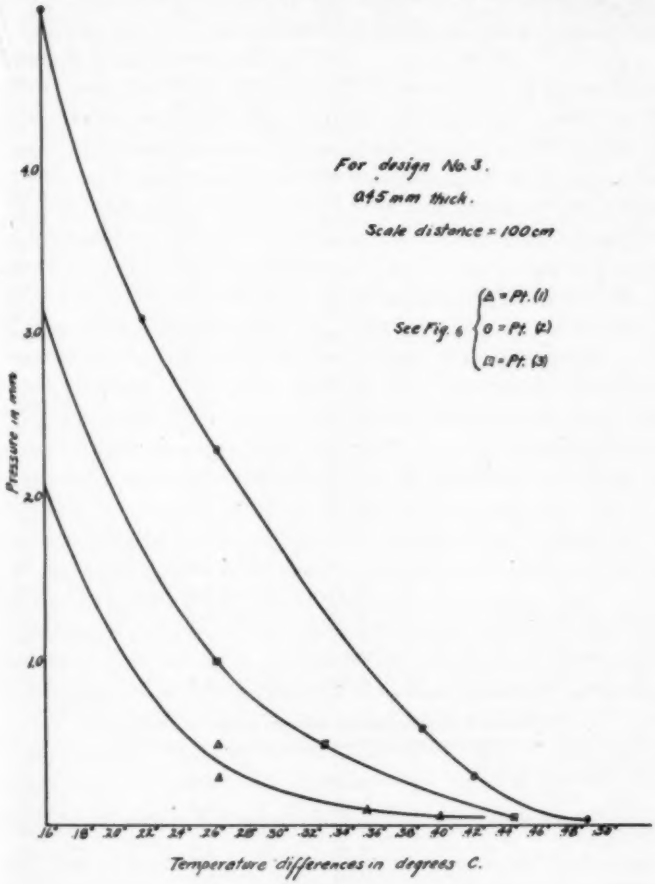

For design No. 3.
.045 mm thick.
Scale distance = 100 cm
See Fig. 6
△ = Pt. (1)
○ = Pt. (2)
□ = Pt. (3)
4.0
3.0
2.0
1.0
Pressure in mm
16° 18° 20° 22° 24° 26° 28° 30° 32° 34° 36° 38° 40° 42° 44° 46° 48° 50°
Temperature differences in degrees C.

FIG. 14.—*Temperature-pressure curves.*

thick and thin vane of design number 3 as a function of the pressure in Figs. 13 and 14. It is apparent that the temperature differences increase rapidly as the pressures decrease, and that the positions 2 and 3 show distinctly greater temperature differences than does position 1.

The increase in temperature difference at position 2 relative to position 3 is due, perhaps, to the fact that the attachment of the suspension near position 2 may have played a role. The greater temperature difference

TABLE 1. Difference in temperature between points (1) and (3)*

Design No.	Thickness mm	Pressure mm	Deflections mm	Temperature Deg. C.
3	0.05	0.5	0.5	0.04
		0.3	1.0−	0.06
		0.5	1.0−	0.06
		0.04	0.5−	0.03
		0.10	0.5+	0.05
		0.06	0.5+	0.05
		0.04	0.5	0.04
		0.25	1.0−	0.06
		0.30	1.0−	0.06
		0.80	1.0−	0.06
1	0.05	0.60	0.5−	0.03
		0.40	0.5−	0.03
		0.40	0.5+	0.05
		0.40	0.9	0.07
		0.30	1.5	0.12
		0.12	0.6	0.05
		0.075	0.6	0.05
		0.50	1.0	0.04
2	0.45	0.85	4.0	0.32
		0.50	4.0	0.32
		0.35	4.5	0.36
		0.30	4.5	0.36
		0.25	5.0	0.40
		0.20	5.0	0.40
		0.10	5.0	0.40
		0.05	5.0	0.42
		0.04	5.5	0.44
		0.03	5.0	0.42
Difference in temperature between points (3) and (4)*				
2	0.45	1.20	2.0	0.16
		0.80	3.0	0.24
		0.40	3.0	0.24
		0.10	3.5	0.28
		0.05	3.5	0.28
		0.04	3.5	0.28

*See Fig. 6.

at position 3 relative to position 1 suggests, perhaps, why the ratio of the deflections, in the case of vanes of larger edge moment ratio, fell so far below the theoretical value since the edges of these had smaller

temperature differences than did the vanes of smaller edge moment ratio.

b. *At different points on the illuminated side.*—Measurements of temperature differences at two points on the blackened illuminated side of vanes of designs 2 and 3 are shown as functions of the pressure in Table 1. Considerable difficulty was experienced in obtaining satisfactory readings for these temperature differences because of the time taken to reach a maximum value.

It will be seen that the solid end of the vane always has a higher temperature at its center than has the edge of the grid end, and that the temperature difference between the two ends varies very little with the pressure. The difference is less for thin vanes than for thick ones which means that the thin vanes should, if properly blackened, follow the edge moment ratio more closely than the thick ones. This did not seem to be true from the earlier measurements but some results obtained in the present work indicate that this is true. Measurements of the temperature differences between the center and the edge of the solid end of the vane show that the edge is distinctly cooler than the center. The temperature differences between the edge and the center of the solid end of the vane is not, however, as great as that between the edge of the grid end and the center of the solid end, thus showing conclusively that the deviation of the ratio of the deflections of the vanes to be expected from the ratio of the edge moments is purely a temperature effect due to the greater cooling at the edges of the vanes. It also indicates the reality of the temperature differences at the edges of the vanes postulated by Hettner and Czerny[3] for radiometers of the vane type.

Calculation of Forces Involved.—The torsional constant of the fibres used was found to be 1.9 dyne-cm. per radian. From this value and the deflection of vane of design 3 observed at 0.03 mm pressure and 40 cm (scale distance), the force acting was found to be 0.02 dynes per cm of edge. For this same vane, using the temperature differences indicated above and the same dimensions, the calculated value of the force from the Einstein equation $\left(F = p\lambda\frac{\Delta T}{T}\right)$ gives the value of 0.03 dynes per cm of edge. Considering that the temperature measurements are of only the roughest type and that the Einstein equation has been deduced to give order of magnitude only (the constant terms being omitted for simplicity), the agreement is surprisingly good.

[3] Hettner and Czerney, ZS. f. Phys., *27*, p. 12; 1924.

CONCLUSIONS

These preliminary rough experiments lead one to the following conclusions. (1) The effective portion of the ordinary vane radiometer in producing the radiometric effects is the narrow region along the edge of the vane for pressures where the mean free path of the molecules does not approach the dimensions of the vane. Consequently the implication of Einstein's theory, that the deflection of the vane should be proportional to the edge moment of the vane rather than to the area moment, seems to be confirmed for cases where the ratio of edge moments is not too great. Where there is a large ratio of edge moments to be compared, the cooling effect of the greater edge length reduces the temperature differences between the two halves of the vane in such a way as to make the greater edge moment less effective in producing radiometric action. (2) The Einstein theory predicts that the deflection should be independent of the pressure in case the temperature differences remain constant. It was found that the sensitivity increased, with the pressure decrease, about fourfold. Very rough temperature measurements as a function of the pressure show that the temperature differences on the illuminated and unilluminated faces of the vane, which is one of the factors determining the deflection, vary in the ratio of one to three for the same range of pressures. It is quite probable that more accurate measurements of temperature will show that the whole change in sensitivity with pressure can be accounted for on a temperature basis. (3) The Einstein equation was deduced in order of magnitude only, and no attempt was made to determine the constant coefficient of the phenomenon. The force per unit length of edge, calculated from this equation, agrees with the observed force in a surprisingly close manner. It seems, therefore, as though the action of the ordinary vane radiometer could be accounted for roughly by the simple Einstein theory.

In conclusion the writer wishes to express his thanks to Professor L. B. Loeb under whose direction this work was carried out.

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JUNE 25, 1925.

DETECTION OF SMALL CHANGES IN RESISTANCE, INDUCTANCE, AND CAPACITY BY MEANS OF AN OSCILLATING CIRCUIT

By L. S. Taylor

The device discussed below contains in itself no new elements—being essentially a Colpitts oscillating circuit of carefully selected constants. Fig. 1 shows the circuit used, and one which may be operated within or above the audible range of frequency, according to the immediate use.

Its operation rests upon the principle that under certain conditions there may be "trapped" in the grid circuit a negative charge of sufficient magnitude to cause the circuit to cease oscillating. Under normal

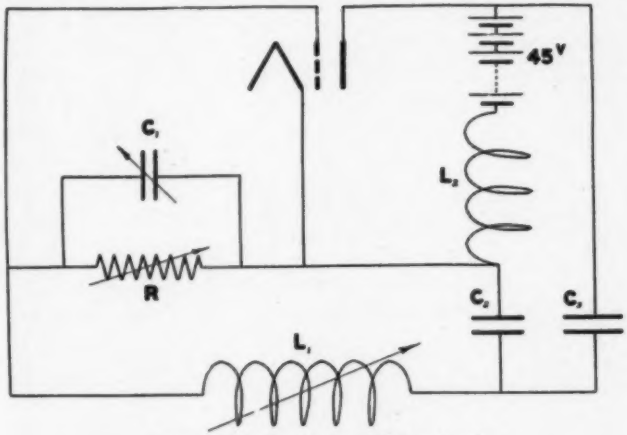


Fig. 1.—*Diagram of the oscillating circuit.*

conditions, the resistance R is such that the grid charge will leak off almost immediately. If, however, the value of this resistance be too large the charge cannot leak off with sufficient speed and will thus be temporarily trapped. However, after a very definite interval of time the charge will escape (provided that R is not "infinite") and the circuit will be again free to oscillate until a trapped charge has again built up, after which the cycle will be repeated. I shall hereafter give the term "beats" to those intervals during which the circuit is oscillating

normally. It is obvious that by a proper choice of R—all other things being kept constant—the frequency of this cycle may be brought to any desired value.

The constants of the remainder of the circuit also play an important part in determining the frequency of this cycle. For instance, as the capacity of the entire isolated grid circuit increases, a greater amount of charge will have to accumulate before the oscillations are cut off. Consequently it will take a longer time for the charge to leak off through R and will, ordinarily, cause a decrease in the frequency of recurrence of the beats. A change in the inductance L_1 has a similar effect, and in general as it is increased, causes a decrease in the beat frequency. By carefully choosing these constants of the circuit also, the beat frequency may be brought within a desired range. Thus any small change in the

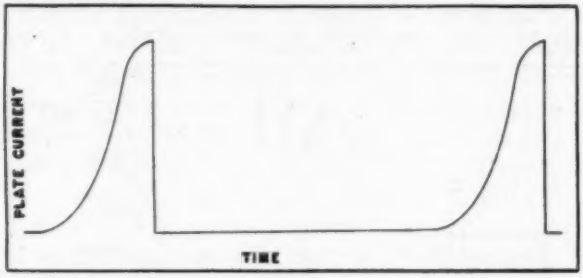


FIG. 2.

resistance, capacity or inductance will cause a corresponding change in the beat frequency, and conditions may be obtained which will bring the working range to a critical part of the characteristic curves as discussed below.

Fig. 2 shows the general way in which the circuit starts oscillating after a silent interval. It is the plate current plotted against time. It will be observed that the cut-off is very sharp, and this suggests at once several means of measuring the beat frequency accurately. In the audible range the break of the tone, as heard in the phones, may be used as a convenient measuring point, and at any frequency the break in the plate current as observed by a galvanometer may be used as the measuring point, thus permitting the use of radio frequencies. The device necessarily measures L_1 and C_1 at the normal frequency of the circuit, and this must be suitably determined in advance by measuring it at the cut-off value under steady oscillating conditions.

Fig. 3 shows the beat frequency per minute as a function of the grid leak resistance R, and at several values of the filament current. In order to measure small changes in R it is necessary to work on the steep portion of the characteristic. This is made possible by adding an extra constant resistance to that being measured sufficient to bring the total to the high value shown.

Beatty and Gilmour[1] have utilized this same general phenomenon and by means of a slightly different circuit have determined resistance values. Their method, although very sensitive, has some slight disadvantages in actual practice—namely: (A) It requires the location of a "transition" point as the factor in determining the resistance, and

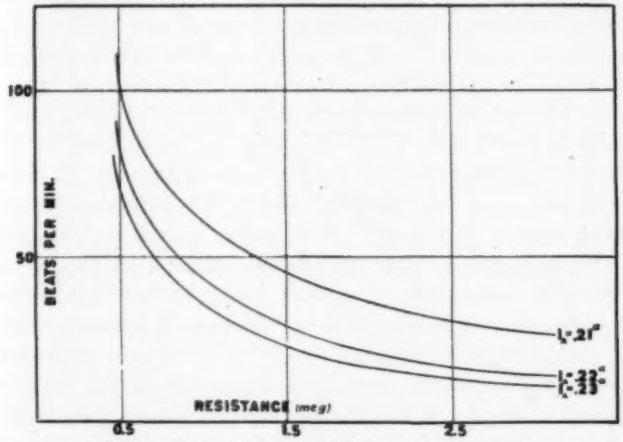


FIG. 3.—*Effect of grid leak resistance on beat frequency.*

this involves separate manipulation of the circuit for each value of the resistance used. (B) It is necessary to adjust the circuit to identical conditions with a known resistance replacing the unknown. (C) The characteristics of the circuit are very involved in the particular region used. (D) A second auxiliary oscillating circuit is required. Using the method herein described, these disadvantages may be wholly or in part eliminated. (A) Using a steep portion of the characteristic curve makes measurements possible over a considerably wider range without any further adjustment of the circuit. (B) Readjustment to identical operating conditions is eliminated since the beat frequency determines at once the resistance or change in resistance. (C) By carefully

[1] Beatty and Gilmour—Phil. Mag., *40*, pp. 291-298; 1920.

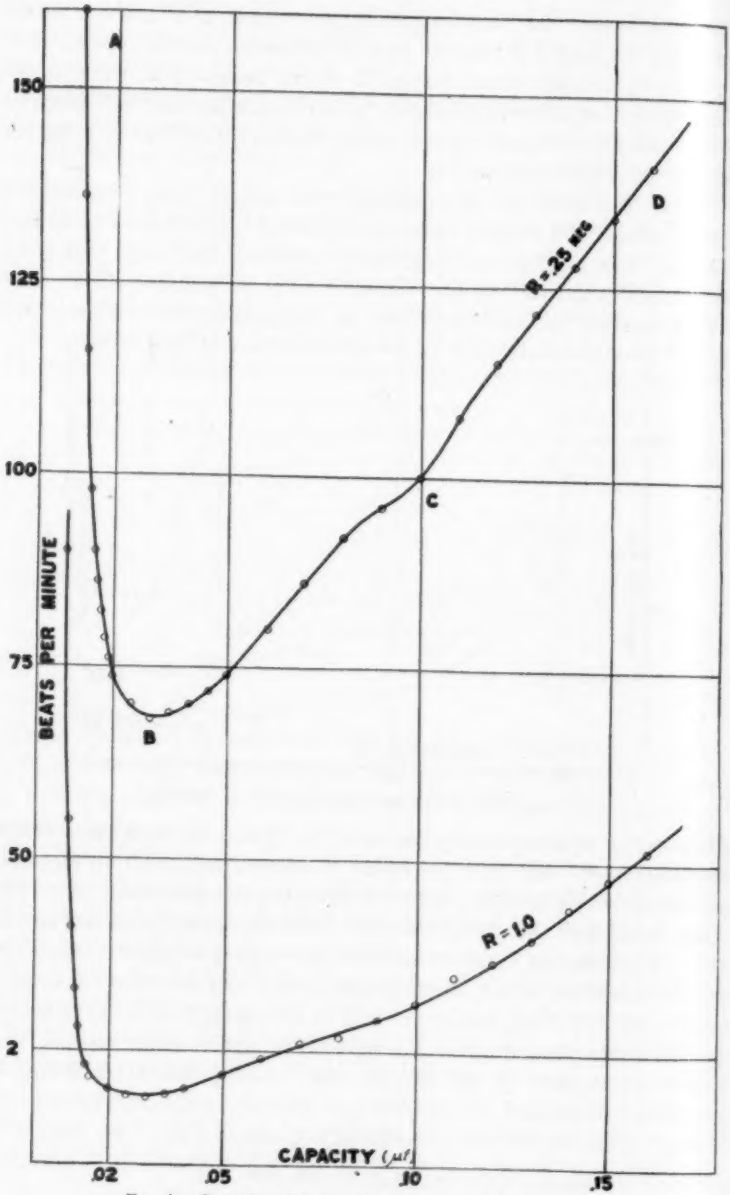


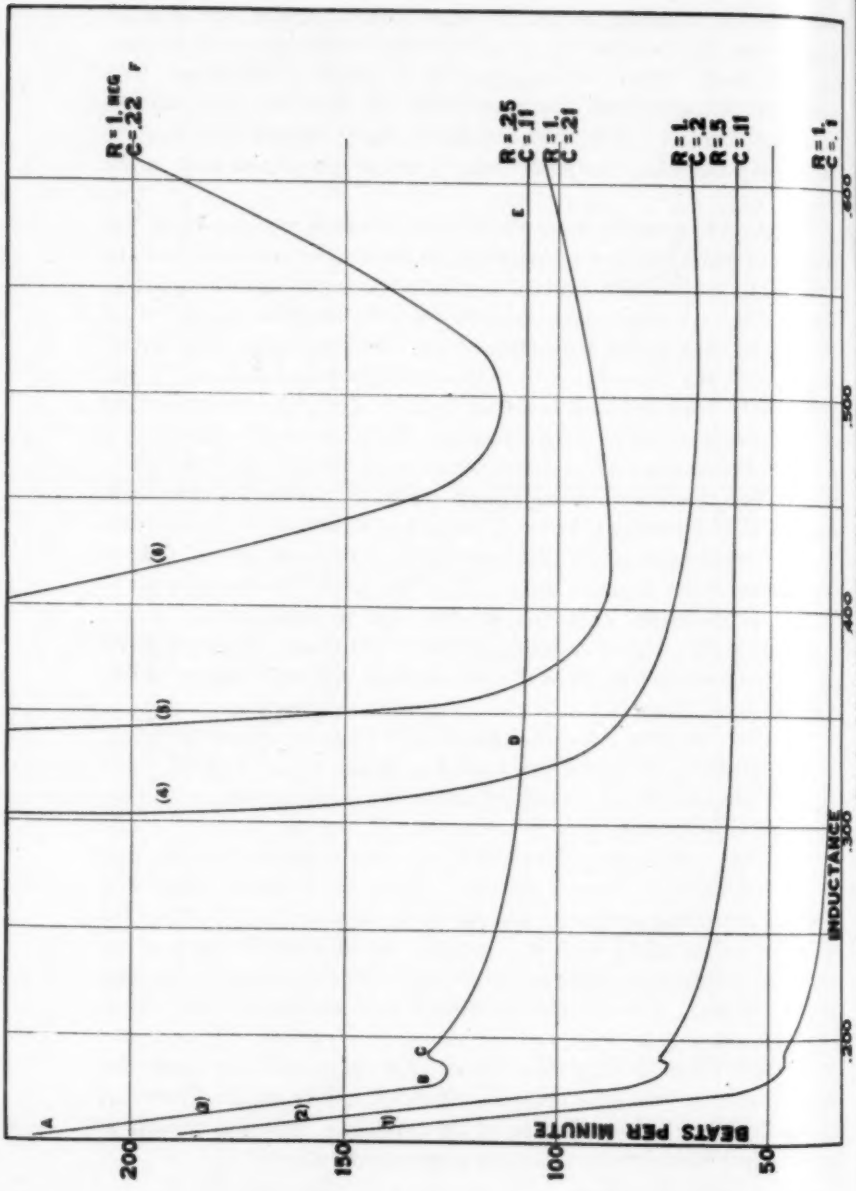
FIG. 4.—*Capacity effect of grid condenser on beat frequency.*

selecting the constants of the circuit, a working region free from irregularities may be selected. (D) The single oscillating circuit is complete in itself. Both methods require a similar calibration. Any unsteady leakage across the tube itself will have no effect on the general operation. The tube resistance, being seldom less than 10 megohms, enters as a reciprocal and so will be a negligible part of the whole.

The effect of capacity upon the beat frequency is more involved and is open to wider range of practical application. The portion AB of the curve (Fig. 4) is of the form to be ordinarily expected and the negative slope being very steep makes possible the determination of a very small change in capacity as a function of the beat frequency. For higher values of C, the curve passes through a minimum and then indicates a very nearly linear relation as shown by BD. The slope of this portion of the curve increases as R decreases, but this is attended by an increase in the beat frequency and a consequent greater difficulty in its measurement. The position of the minimum value B varies but little with changes in R, although it becomes broader as R increases. The position of the steeper portion AB also varies little, and so in general it is of advantage to use higher values of R. The breaks in the curve at C appear to definitely exist and are not due to experimental errors. A theory of the effect of capacity on the beat frequency has been given by Rschewkin and Wedensky[2] for a slightly different circuit, but it does not seem to apply to the effects found in this study.

The effect of inductance changes is more pronounced and probably more applicable, as will be seen from Figs. 5 and 6. Here again it will be desirable to work on a steep portion of the characteristic, this being made possible by adding more inductance or varying other constants. In Fig. 5 two sets of curves are given for the beat frequency as a function of the inductance. One set (curves 1, 2, 3) is for different values of R with C_1 remaining constant, and the other set (curves 1, 4, 5, 6) for different values of C_1 with R constant. By changing R there is no essential change introduced in the shape of the characteristic, the two major portions, AB and DE remaining very nearly parallel. As R increases there is a slight shift of the steeper portion AB to higher inductance values, but the principal effect of varying R is to change the beat frequency over wide ranges. No explanation can be offered at present for the double inflection of the curve at C, which is present to some extent in every characteristic at the lower values of L_1.

[2] V. S. Rschewkin and B. W. Wedensky. Phys. Zeit, *23*, pp. 150-152; 1922.


R = 1 MEG
C = .22 mf
R = .25
C = .11
R = 1.
C = .21
R = 1.2
R = .5
C = .11
R = 1.
C = .1
(5)
(3)
(4)
E
D
C
A
(1)
(2)
(3)
200
150
100
50
BEATS PER MINUTE
.600
.500
.400
.300
.200
INDUCTANCE

By fixing R and changing the inductance for different values of C_1 the curves assume a more complicated character with an increase in capacity. For lower values of C_1 the inductance change over a considerable range has no effect whatever on the beat frequency within the beats themselves. However, as C increases the formerly horizontal portion of the curve no longer remains linear but takes on a changing positive slope which after a certain point increases very rapidly with an increase in C. Thus in curve 6 the positive and negative slopes of the major portions of the curve are approximately numerically equal. When using a curve in this position there is no advantage in one portion over the

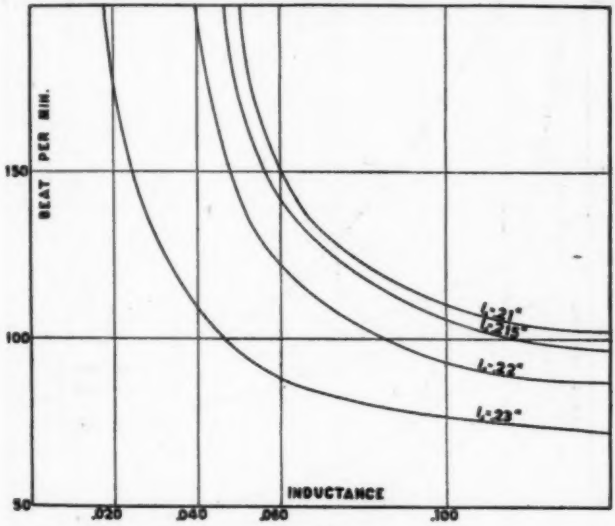


FIG. 6.

other, when some distance from the point of inflection. A very important use of the capacity control is to vary the position of the inductance range over which it is necessary to work. However this method of control is accompanied by the disadvantages of decreasing the sensitivity to very small changes or necessitating the measurement of much higher beat frequencies.

As the filament current I_A, is increased (Fig. 6) a greater number of electrons pass to, and are trapped by, the grid circuit thus consuming a greater time in leaking off through the resistance R. This increases the period of silence and consequently decreases the beat frequency. For smaller shifts in the working range, the adjustment of the filament

current serves as a more useful control than the capacity. By this means the sensitive limits of inductance may be varied without increasing the beat frequency to any great extent.

Since the whole beat effect is also dependent upon the electron emission, great care must be taken to keep this constant. This control will of course vary with the particular type of vacuum tube used, although it was found that all the tubes tried gave the desired effect.

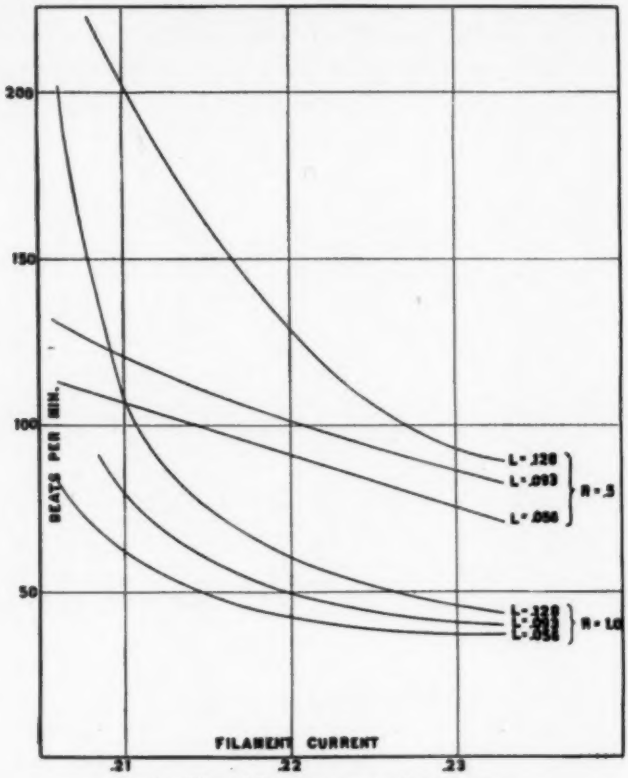


FIG. 7.—*Effect of filament current on beat frequency.*

Fig. 7 showing the effect of electron emission at various values of inductance and resistance is probably not of any great practical value in itself, but serves to show how it may be used as a regulation of the beat frequency. Thus by using a small filament current and comparatively large inductance within a limited range, it is possible to

work on the steep portion of that curve. This agrees with the results as observed from Figs. 3 and 6, and serves as an aid in choosing the working range.

Several interesting and perhaps useful phenomena occurred in connection with this study. At very high beat frequencies a very definite and distinct point is reached where the beats occur at such

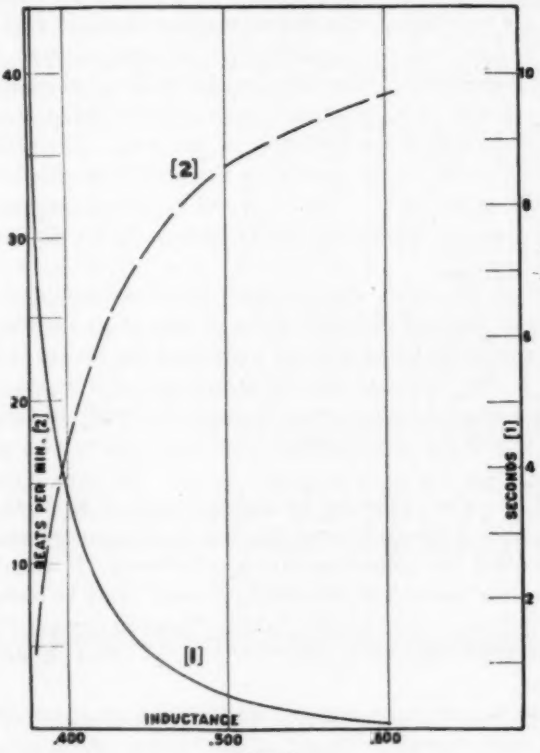


FIG. 8.

great frequency that the tone is no longer "throbbing" or harsh, but becomes the pure tone of the natural oscillator. Suppose then, that the circuit is giving beats just below this value and the inductance be decreased sufficiently to give an even tone. The change occurs instantly. However, upon restoring the inductance to its former value the circuit will not commence to beat again until the inductance has been considerably further increased. Under some conditions it may resume

beating at the former inductance value, but only gradually and after a long period of time. The same effect exists at any high beat frequencies to a degree decreasing with the beat frequency. There is thus a definite time lag in one direction, in which the circuit assumes its steady beat conditions. The same general effect also occurs when the capacity or filament current is varied in the proper direction.

Another point of interest occurs when R is large and C_1 small so that the beats are very slow. Conditions may be obtained such that the length of the beat greatly exceeds that of the silent interval. The start of the beat is preceded by a definite muffled click in the phones, and is closed by a similar click. It would suggest the possibility of some very definite valve action in the formation of the beats. The clicks cannot be heard for the very sharp beats since the time interval is too short in which to distinguish them. Fig. 8 shows curves of the length of the beat (1) and the corresponding beat frequency (2) for different values of the inductance.

All of the curves shown were obtained within the audible range, and the beats were counted as heard in the phones of an auxiliary circuit. By working at radio frequencies and counting the breaks in the plate current, it is observed that the characteristics are all very steep and so very much smaller changes may be easily detected. In actual trials, the circuit was found to hold very steady and results were accurately repeated.

A circuit was also calibrated by known values of R, L_1 and C_1 and when other known constants were introduced, their corresponding beat frequencies fitted the calibration curves very closely. It was also found that after several changes in the circuit, it could easily be brought back again to conditions proper to fit it to its calibration curves. The age of the vacuum tube appears to have little or no effect in shifting the calibrations.

The study is not yet completed and after further information is obtained a more complete theory of the various phenomena will be discussed.

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A CAMERA FOR PHOTOGRAPHING THE INTERIOR OF A RIFLE BARREL*

By I. C. Gardner and F. A. Case

The camera here described was developed for use in connection with the study of the erosion of the barrel of the .30 caliber machine gun or service rifle. For a large gun a study of the progress of the erosion during a firing test is made by means of wax impressions of the interior of the barrel which are made from time to time during the test. For the .30 caliber barrel this method is not convenient because of the smallness of the bore. Accordingly it has been the practice to split the barrel lengthwise when it is desired to examine the condition of barrel. This has the obvious disadvantage that after the examination of the bore the barrel is no longer suitable for a continuation of the test. The history of the erosion in a single barrel therefore cannot be followed but it is necessary to make assumptions regarding the uniformity of several different barrels fired for different numbers of rounds in order to interpret the progress of the erosion. To escape these difficulties it was decided to build apparatus to photograph the interior of the bore.

An apparatus for this purpose has been built by Nakamura.[1] In this instrument the entire periphery of the bore for a short length is imaged by a microscope objective coaxial with the barrel and the image

Fig. 1. *Diagrammatic sketch of camera.*

is projected through the length of the bore by the usual periscopic system upon the film placed at the end of the barrel. The disadvantages of this system lie in the presence of excessive fore-shortening and consequent lack of uniformity of scale over different parts of the photograph. It was decided not to adopt this plan but to devise an optical

* Published by Permission of the Director of the National Bureau of Standards of the U. S. Department of Commerce.

[1] S. Nakamura, Japanese Journ. Phys., *1*, p. 21; 1922.

system by which the cylindrical inner surface of the bore is truly developed in a plane without fore-shortening or distortion.

The method of accomplishing this is made clear by the diagrammatic sketch of the instrument shown in Fig. 1. A periscope is built of such dimensions that it can travel the length of the interior of the barrel. This is shown at *P* partially inserted in the rifle barrel *B*. The periscope is of unit magnification and projects an image of a small portion of the interior of the bore on the motion picture film at *F*. The barrel of the rifle is slowly drawn lengthwise along the tube of the stationary periscope thus bringing different portions of the bore successively into the field of view. This causes the image to move on the motion picture film. As the periscope is of unit magnification, if the film is unrolled at the same linear speed of travel as that of the gun barrel the image

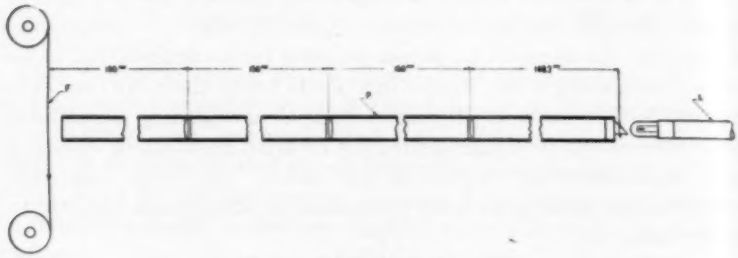

FIG. 2. *Details of optical system.*

will be stationary relative to the film and as a result of the travel one obtains on the film a long strip exposure carrying the picture of a longitudinal strip of the bore. The width of such a strip is approximately one sixth the complete circumference of the bore. Consequently by taking six such pictures with the barrel rotated an appropriate amount between each exposure one obtains photographs of the entire surface of the bore. The synchronous motion of the barrel and film is maintained by suitable gearing.

The chief difficulties arise from the necessary smallness of the parts which must enter the barrel of the rifle. Fig. 2 is a diagrammatic view of the periscope. The mounting for the lenses consists of an outer brass tube having a .02 inch wall thickness. The lenses are fitted into this tube and kept suitably separated by spacing members which are lengths of tubing which telescope into the outer tube. The wall thickness of the spacing tube is also .02 inch. The optical system consists of the objective prism which bends the rays to pass along the axis of

the tube and three identical achromatic doublets, focal length 75 mm which are spaced 150 mm apart. The first lens of the system is the objective and as the conjugate distances are each $2f$ there is formed in the plane of the second lens an inverted image of natural size of the interior of the bore. The second lens is a collector lens and as it is in the plane of the image produced by the first lens it does not alter the magnification. Ordinarily this lens would have a single component as is usual for collector lenses but the doublet is used as it seems simpler to make all the lenses of the system identical. For the third lens the

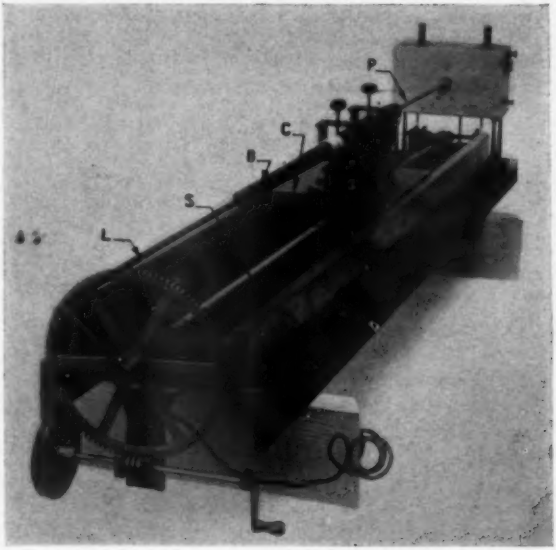

FIG. 3. *View of complete camera.*

conjugate distances are also $2f$ and it therefore projects on the film an image of the interior of the bore in natural size. As a source of illumination a small low voltage electric lamp supported by a tube introduced from the muzzle end of the barrel is employed. For a single exposure with barrel and film stationary the area exposed on the film is 4×1 mm with the 1 mm dimension extending in the direction of travel of the film. During the progress of a continuous strip exposure the barrel travels at the rate of approximately 1 mm per second and consequently the length of exposure for any one point is approximately 1 second.

The details of construction are clearly shown in the two photographs of Figs. 3 and 4. The barrel of the rifle *B* is shown mounted on two supports on the carriage *C* which moves along the ways. Below the carriage is the screw *S* which is clearly visible and which has at its outer end a 100 to 1 worm reduction gear. A grooved pulley enables one to drive the apparatus by a small motor and a crank on the other end of the shaft bearing the worm permits the screw to be turned by hand. Turning the apparatus by hand has been found satisfactory in most cases, but for long exposures the use of a motor will be necessary. The screw is of steel, diameter 3/4 inch, with double thread, 1/4 inch

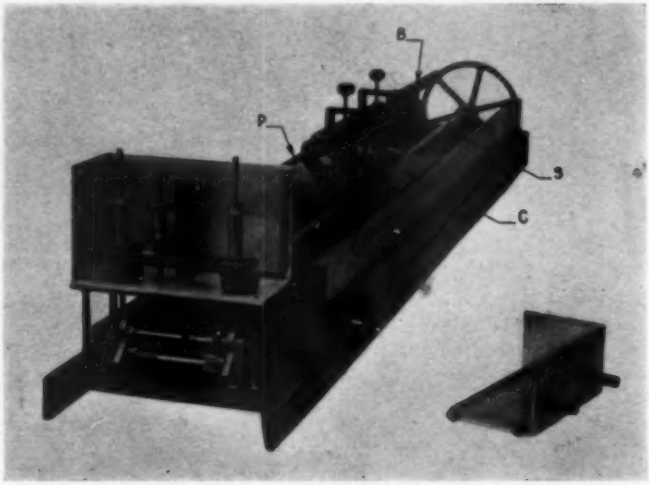

FIG. 4. *View of camera showing film rolling mechanism.*

lead. The carriage is connected to the screw by a split nut which can be released for rapid traversing of bore of barrel. The gear at the end of the worm connects with the film rolling mechanism. The film used is standard motion picture film carried on two spools with a spring take-up device to hold the film taut. A sprocket from a motion picture machine engages in the perforations of the film and causes it to move at the proper rate. The tube *P* inserted in the barrel from the breech end is the periscope. It must be so set that the plane of reflection of the objective prism is parallel to the direction of travel of film as otherwise the travel of the gun barrel will impart a motion to the image with a component normal to direction of travel of film and the photograph

will be blurred. The miniature lamp is carried on the tube *L* inserted from the muzzle end of the barrel and receives current from a small transformer which is fed from the lighting circuit and which delivers 6 volts to the lamp.

This apparatus is useful, not only for photographing the interior of the barrel, but also may be used advantageously for the visual inspection of any portion of the bore. The periscope has only unit magnification, but the definition is sufficiently good to permit one to examine the image in the focal plane with a microscope magnifying 20 to 40 diameters. One can perhaps examine the barrel better this way than

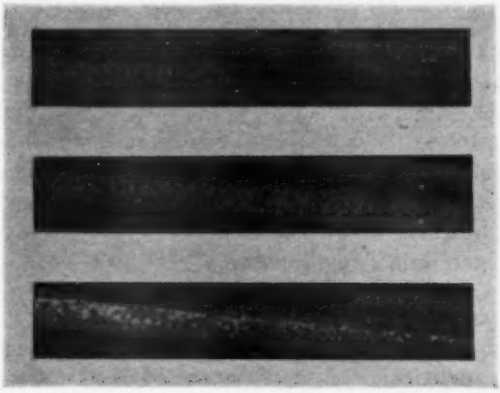

FIG. 5. *Photographs made with gun barrel camera.*

These photographs show the interior of the bore of a badly eroded 0.30 caliber machine gun barrel. The actual dimensions of the area shown in each photograph are 4.7 × 25 mm.

Upper photograph: exposure made near the breech.

Middle photograph: exposure made near middle of barrel.

Lower photograph: exposure made near muzzle of barrel.

photographically as subsequent magnification of the film may give trouble because of grain size in the negative, and furthermore in the visual examination the colors provide much more contrast than is afforded by the black and white of the photograph.

Fig. 5 shows three photographs taken with the camera. The barrel photographed was a badly eroded machine gun barrel. The actual dimensions of the area shown in each photograph are 4.7 × 25 mm and the width of each exposure corresponds to approximately one sixth of the circumference of the bore. The first exposure is taken near the breech, the second midway along the length of the barrel, and the

third at the muzzle end. The lands and grooves can be clearly distinguished, somewhat inclined to the horizontal because of the twist. In the first exposure the lands are almost worn away and the bore is filled with small cracks resembling those in crazed china. The photographs taken farther from the breech show the lands more distinctly and a great deal of erosion in the bottom of the grooves. Viewed visually through a microscope and the periscope, the different characteristics are differently colored and much more detail is shown than can be seen on the photographs.

This is the first instrument that has been built and is to be considered as a well developed model showing the possibilities of this method. If a second instrument were to be made, a better method of supporting the periscope tube would be adopted and more precise gearing would be used to connect the motion of film with motion of barrel. In this way a sharper picture could probably be obtained. Also the gun barrel would be rotated as it travels lengthwise in such a manner that the field of view of the periscope traces a spiral which includes the entire surface of the bore. The film will then be displaced in such a manner that one would secure the entire interior of the bore developed into a rectangular figure in a single exposure.

BUREAU OF STANDARDS,
WASHINGTON, D. C.

Ocular Therapeutics. A Manual for the Student and Practitioner. By Doctor Ernst Franke, Professor of Opthalmology at the University of Hamburg; translated by Clarence Loeb, A.M., M.D., Head of the Department of Opthalmology of the Michael Reese Dispensary, Chicago. 183 pages. C. V. Mosby Company, St. Louis, $3.50.

In this compact volume the author has surveyed all of the methods of ocular treatment aside from optical and operative methods and listed them under three headings—general treatment, local treatment, treatments of special parts. The well organized index aids greatly in turning to the subjects sought and there one finds a wealth of suggestions. Many of the suggested preparations are of German make and rarely found in the average American drug store but there are usually several substitutes suggested and many of these are available. The selection of the method of preference the author leaves to the reader. Though this epitome is primarily intended for the practitioner it should find an equally useful place in the hands of the oculist.

D. F. SMILEY

A NEW ELECTRICAL RECORDER

By Thomas R. Harrison

During the past five years the Brown Instrument Company of Philadelphia, Pa., has developed a recording and automatic signalling and controlling instrument of the deflecting D'Arsonval galvanometer type for use with thermocouples, radiation pyrometers, resistance thermometers, thermal conductivity gas analysis apparatus and various other devices mentioned later. When used with gas analysis apparatus the ammeter, adjusting rheostat and slide wire for zero adjustment are mounted inside of the recorder case.

The usual method is adhered to of allowing the pointer of the instrument to swing freely above the chart except when periodically depressed for making a record. Numerous mechanical improvements are incorporated which combine to give a high degree of accuracy, very distinct records, ruggedness of the instrument and simplicity and ease of its care and operation. Fig. 1 shows external view of the instrument, the case of which is a black enameled aluminum die casting measuring fifteen inches high, fourteen inches wide, and nine inches deep. The chart of this instrument is seven inches wide and when traveling at the standard rate of one inch per hour the height of the instrument permits the record for the past twelve hours to be visible. Various other chart speeds are obtainable ranging from one-quarter inch to six inches per hour; interchanging two gears with which the standard instrument is provided gives a paper speed of four inches per hour.

The instrument shown in Fig. 1 makes either a single chart or by use of an automatic switch it makes records from two, three, four, or six thermocouples, distinguished by the different colors and color combinations in which the records are printed. A duplex instrument, which has two galvanometers, and two charts printed on one strip of paper occupies a somewhat wider case. By using a duplex instrument with selective switches a twelve point recorder may be obtained. The chart passes over a rectilinear knife-edge which is at right angles to the direction of travel of the paper and directly above the paper feed roll. A carbon or inked ribbon is stretched across the paper directly above this knife-edge. The galvanometer pointer swings above the ribbon and when the depressor bar drops from its position above the pointer and the knife edge it strikes the pointer causing a dot to be imprinted

from the inked ribbon onto the chart at the point where the pointer crosses the knife edge. In this way rectilinear coordinates are readily obtained.

With the standard arrangement of one depression each thirty seconds the dots in each record are close enough together to render an almost continuous line. The inked ribbon feeds from the spool, across the chart and onto another spool. A chart and spool each last two months. New

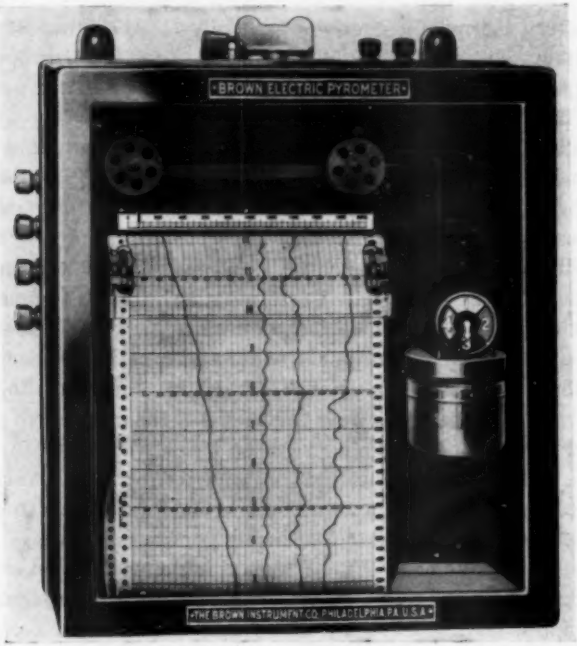

FIG. 1. *New Brown recording pyrometer.*

ribbon spools are easily slipped into place and no re-inking is required By placing the inked ribbon above the chart the mark is made on the front of the chart, which is therefore much more clearly defined and distinct than when the mark is made on the back of transluscent paper. The marking ribbon of the single and duplex recorder, after each mark on the chart, is moved back disclosing the last impression so that the record is clearly visible immediately after it is produced. (Fig. 2.)

An automatic selective switch is used with the multiple recorders which consists of gold contacts mounted on bakelite and immersed in oil. The oil keeps the contacts perfectly clean and in good electrical condition at all times. The switch is provided with a dial marked with the various numbers of the thermocouples and showing the color or color combination corresponding to each as used on the record

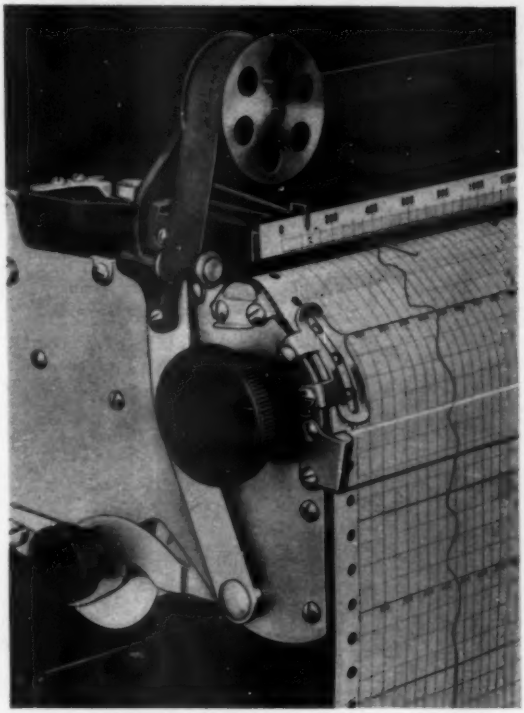

FIG. 2. *Marking ribbon moved back to make record visible.*

chart. A hand rotates on the dial showing which thermocouple is connected to the recorder at any instant (Fig. 1). A platen placed directly below the driving roll is convenient for making notes on the record chart with pen or pencil (Fig. 3). A glass knife-edge for tearing off the paper is placed directly below the driving roll, permitting the record to be torn off up to within two hours of the time the record is made. For example, a complete record for the previous twenty-four

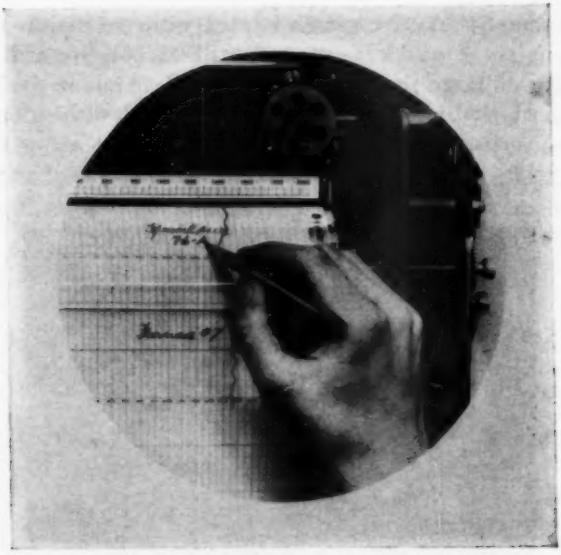

FIG. 3. *Support for writing notes on chart.*

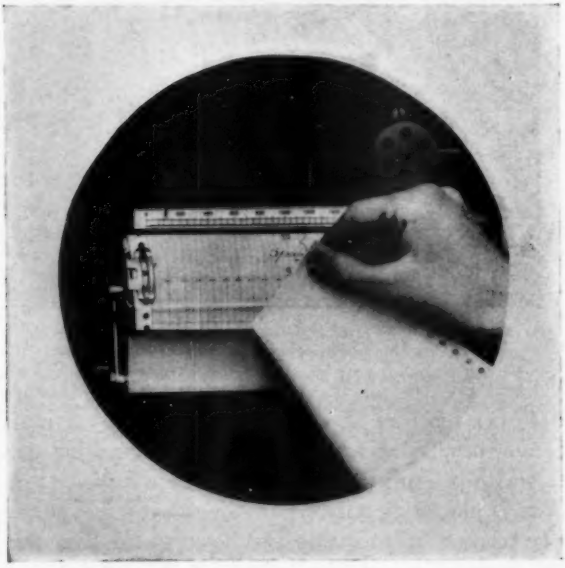

FIG. 4. *Glass tear-off for chart.*

hours and ending at 8:00 a.m. can be torn off at about 10:00 a.m. (Fig. 4.). The galvanometer and recording chart mechanism are carried on a hinged frame which when swung aside makes the galvanometer immediately accessible and when closed provides a housing which gives the galvanometer exceptionally good protection. The zero adjusting

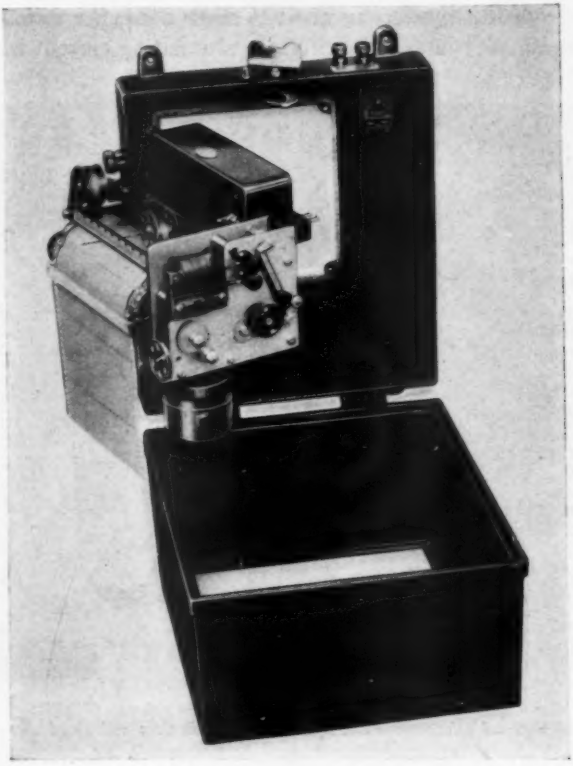

FIG. 5. *Recorder opened up to show assembly.*

screw is carried on this frame and is within the case secure from tampering when the case is closed. (Figs. 5 and 6.) The recorder carries an indicating scale provided with large figures which are legible at a considerable distance.

For use with thermocoulpes the recorder is provided with a bimetallic spring cold junction compensator, and a bimetallic index which indi-

cates the correct "zero" or open circuit reading for the compensated instrument. The moving element has from 50 to 80 ohms resistance and the total resistance is from 180 to 700 ohms according to the range of the instrument and the kind of thermocouple used.

A re-roll attachment is furnished where desired to roll up the chart automatically over long periods, though for twenty-four to forty-eight hour periods it is more convenient to allow the paper to fold itself up in the lower part of the case. Where alternating current is available,

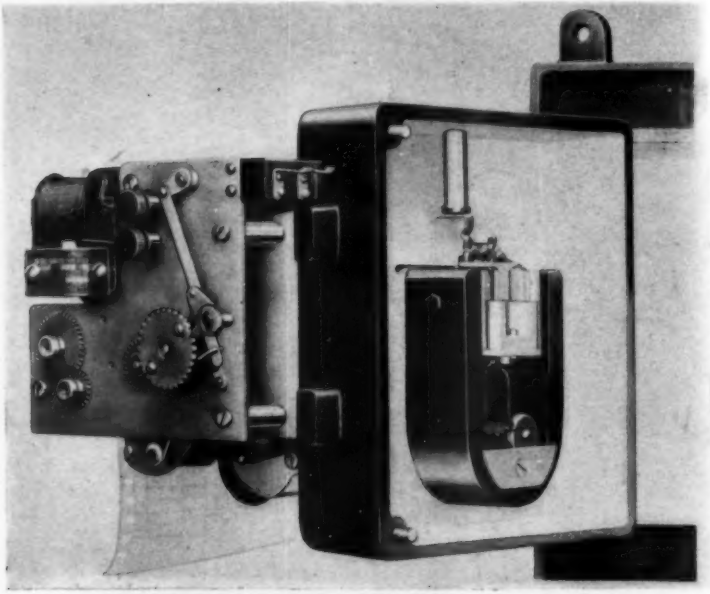

FIG. 6. *Enclosed galvanometer and chart driving mechanism with speed change gears*

the recorder is operated by a four watt synchronous clock motor, thus eliminating the winding of clocks, use of governors, etc., for securing accurate time keeping qualities. By the use of the oil switch described above and other refinements these clock motors can operate the mechanism with ease. This greatly simplifies the entire mechanism and makes it unusually dependable, and with the charts and inked ribbon lasting two months makes the instrument remarkably easy to operate. In the absence of alternating current, hand wound clocks are used.

These recorders can be supplied equipped with the Heatmeter circuit[1] and miniature rheostats equal in number to the switch points on the selective switch. One rheostat is connected in series with each thermocouple circuit. Pressing the push button shows whether each rheostat is properly adjusted to calibrate the instrument for inequalities in circuit resistance caused by various lengths of lead wires, etc. No change in recorder reading when the button is pressed indicates a correct

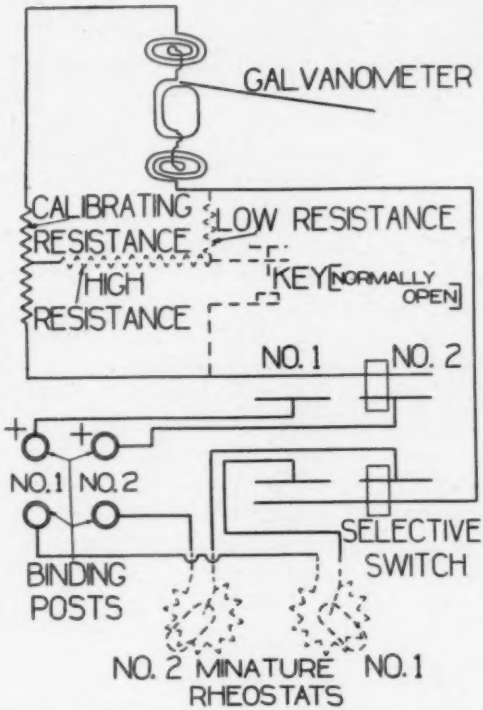

FIG. 7. *Wiring diagram of two-point recording "heatmeter."*

rheostat adjustment, independent of contact resistance, etc. Very infrequent readjustments of the rheostats are usually sufficient but on account of the ease of making the test it is never necessary to make assumptions as to the correctness of the circuit resistance.

On account of the principle of magnification of errors for making corrections, an adjustment which is apparently good to five degrees

[1]Harrison and Foote, Jour. A.I.E.E., Feb. 20, 1920; U. S. Letters Patent 1,565,093.

will actually be good to one degree, and so on in proportion. The wiring diagram of a two-point recording heatmeter is shown in Fig. 7 the parts drawn in solid lines being those common to all recorders of this type and the dotted parts being the additional parts involved in the heatmeter circuit. Where the rheostat contacts may be affected by corrosive gases in the atmosphere they can be supplied in cases suitable for filling with oil which may be mounted within the recorder case or outside on a panel.

A controlling or signalling attachment has been developed for this instrument which is capable of handling larger currents than usual, eliminating the necessity of relays in many cases. The contacts for this attachment are operated by power derived from the driving motor. When the galvanometer needle is depressed the appropriate one of several pivoted arms is swung out of a position of rest. A cam then allows the switch to move forward until stopped by the displaced arm, thus fixing the position of the switch in accordance with length of the chosen lever. In different positions the switch makes different connections. The pivoted arms are of various lengths to select the different switch positions. Different arms are thrown out by the pointer depressing against different spring extensions spaced adjacently to correspond to the usual high, neutral and low positions. Even more pivoted arms and switch contacts can be supplied where desired. In addition to use as pyrometer recorder, gas analysis recorder, etc., the instrument may be provided with suitable sensitive elements and be used as a recording voltmeter, ammeter, and tachometer when attached to suitable direct current generator and for many special applications. Several of the features of this new recorder described in this article are covered by seven pending patent applications.

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